Computable General Equilibrium Models:
Factor supply

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Simulation Models for Policy Analysis

Summer Term 2020

Aims for today

- Learn about the CET-function as the sister of the CES
- See how the CET-function can be used to depict factor supply
- Reflect on factor quality, prices and mobility

CET function

- Constant Elasticity of Transformation
- Functional identical to CES
- Where CES aggregates inputs to output, CET distributes (= transforms) a given total quantity to different qualities
- Dual price index and optimal transformation quantities derived from revenue maximal distribution
- Can also be understood as a non-linear aggregator of different (factor) qualities

CET function

average factor price

sector specific factor prices

```
v_pf(r,f)  =e= sum(a, p_gf(r,f,a)*v_pfa(r,f,a)**(1+p_omegaf(r,f)))**(1/(1+p_omegaf(r,f)));
```

revenue share
In benchmark

price relation exponent
transformation elasticity

```
v_xf(r,f,a) =E= p_gf(r,f,a) * p_xf(r,f) * (v_pfa(r,f,a)/v_pf(r,f))**p_omegaf(r,f);
```

factor supply to sectors:
defines sector specific
factor price *v_pfa*

total factor supply

CET: no mobility

Linear price aggregator

```
v_pf(r,f)  =e= sum(a, p_gf(r,f,a)*v_pfa(r,f,a)**(1+p_omegaf(r,f)))**(1/(1+p_omegaf(r,f)));
```

If p_omegaf = 0, i.e. no mobility

Exponent term are 1

Last term is 1

```
v_xf(r,f,a) =E= p_gf(r,f,a) * p_xf(r,f) * (v_pfa(r,f,a)/v_pf(r,f))**p_omegaf(r,f);
```

Fixed shares of sector on given total stock => Fixed sectoral factor stocks

CET: Full mobility

- Full factor mobility implies (elasticity is infinite):
  - Factor is fully homogenous, i.e. no quality differences between sectors
  - Linear (!) aggregation

```
e_pfLin(r,f) $ (p_omegaf(r,f) eq inf) ..
   p_xf(r,f)  =e= sum(s, v_xf(r,f,s));
```

  - Law of one price, i.e. factor prices are identical across sectors

```
e_pfaLin(r,f,a) $ (p_omegaf(r,f) eq inf) ..
   v_pfa(r,f,a) =E= v_pf(r,f);
```

CET and time horizon

- Remember from your micro-economic classes the differences between short-run and long-run supply
- In a CGE, these differences can be reflected in the factor transformation elasticity:
  - Very short run: factors are immobile (elasticity=0)
  - Medium term: higher than zero, but not infinite (e.g. part of capital is depreciated)
  - Long term: full factor mobility

CET and other aspects

- GTAP data base comprises "natural resources" (marine fish stock, crude oil, natural gas, coal, gold, rare earth …) as a primary factor
- Does not make sense to consider these mobile (the crude oil industry cannot replace crude oil with marine fish stocks):
  - Natural resources are considered immobile
- GTAP data base comprise land (only in agricultural sectors and forestry) as primary factor, here, as a default, limited mobility ("sluggish") is assumed

CET nestings

- Similar to nested CES production function, nested CET structure are possible
- Used e.g. in MAGNET

Allocation land over sectors:
Moving land from one use to another involves adjustments costs. To capture this, land is treated as a sluggish input in the GTAP model. A nesting structure was developed for the CET function to allow for differences in the ease of land use change between different land use types. Also the possibility of perfect competition on the land market is made available. This module offers two alternative options for land allocation: CET allocation treating land as sluggish with more nests than in the standard GTAP model and treating land as a perfectly mobile endowment.

http://www.magnet-model.org/modules.aspx

CET nestings in CGEBox

- Generic approach, i.e. equations in model code which can work with different nestings

```
*
*  ---- CET supply sub-nests (not part of standard GTAP model)
*
pfNestEq(rs(r),fm,fNest,ts(t)) $ gfNest(r,fm,fNest,t) ..

   0 =e=
*
*        --- sluggish factor mobility, dual price aggregator for sub-nests
*
       (pfNest(r,fm,fNest,t)
*
*            --- factors linked into sub nest
*
             - [    sum(fNest_a_f(fNest,a,fm) $ gf(r,fm,a,t),
                        gf(r,fm,a,t)*pf(r,fm,a,t)**(1+omegafNest(r,fNest,fm)))

*
*            --- subnests linked into sub nest
*
                 +   sum(fNest_n_f(fNest,fNest1,fm) $ gfNest(r,fm,fNest1,t),
                          gfNest(r,fm,fNest1,t)*pfNest(r,fm,fNest1,t)**(1+omegafNest(r,fNest,fm)))
               ]**(1/(1+omegafNest(r,fNest,fm)))

             ) $ (omegafNest(r,fNest,fm) ne inf)
*
*        --- fully mobile factors, uniform prices => physical aggregation
*
      +  (xfNest(r,fm,fNest,t)/xft.scale(r,fm,t)
                    - sum(fNest_a_f(fNest,a,fm) $ xfFlag(r,fm,a), xf(r,fm,a,t))/xft.scale(r,fm,t)
                    - sum(fNest_n_f(fNest,fNest1,f),              xfNest(r,fm,fNest1,t))/xft.scale(r,fm,t)

         ) $ (omegafNest(r,fNest,fm) eq inf)
      ;
```

CET nestings in CGEBox

- Generic approach, i.e. equations in model code which can work with different nestings:

```
*
*   --- new capital is fully mobile
*
    fNest("newCap")                          = YES;
    fNest_a_f("newCap",a,"capital")          = YES;
    fNest_n_f("xft","newCap","capital")      = YES;
    omegafNest(r,"newCap","capital")         = inf;
*
*   --- old capital is fixed
*
    fNest("oldCap")                          = YES;
    fNest_a_f("oldcap",a,"capital")          = YES;
    fNest_n_f("xft","oldCap","capital")      = YES;
    omegafNest(r,"oldCap","capital")         = 0;
*
*   --- Leontief for distribution of total capital
*
    gfNest(r,"capital","oldCap",t)   = (1-fdepr(r,t));
    gfNest(r,"capital","newCap",t)   = fdepr(r,t);
```