Journal of Global Economic Analysis, Volume 5 (2020), No. 2, pp. 63-109.

Land Use in Computable General Equilibrium Models

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Constant Elasticity of Transformation (CET) functions are widely used to allocate land across uses in Computable General Equilibrium (CGE) models. These models fail to maintain the physical area of land in balance. This paper examines this issue. It shows that heterogeneity in land prices (rents) is the main source of imbalance in land area, not the curvature of the CET function. It also shows that the available approaches that restore balance to physical area either introduce ad hoc adjustments in land allocation or undermine the conventional welfare assessments of the CET results. An alternative approach involves implementing stochastic productivity distribution functions (e.g. Fréchet distribution) to allocate land among uses maintain area of land in balance, thereby respecting conventional welfare assessments. A particular feature of these models is that the aggregate production functions of the land using sectors exhibit decreasing returns to scale even if land is the only factor of production. This approach also requires equalization of land rents across uses. This is not consistent with empirical observation. Both the CET and stochastic methods consider the implicit opportunity costs of moving land across uses but fail to take into account preparation costs associated with land use conversion.

JEL codes: C68, Q15, Q24, I31.

Keywords: Land use; CGE; CET; Cost of land transformation; Welfare impacts.

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1. Introduction

During the past two decades major attempts have been made to augment Computable General Equilibrium (CGE) models to provide analyses of induced land use changes due to human activities and examine land use implications of climate change and associated policies (for details see Hertel et al. (2009) and van Tongeren et al. (2017)). However, incorporating land use in a CGE model is not an easy practice. Several approaches have been developed to accomplish this task[1]. Following the initial work developed by Powell and Gruen (1968)[2], many CGE models apply Constant Elasticity of Transformation (CET) functions to allocate land across alternative uses[3]. This approach takes into account the fact that the quality of land varies across uses and that moving land among uses is costly[4] (van Tongeren et al., 2017). These are the most important determining factors in modeling land use change. However, the CET approach fails to maintain the physical area of land in balance (Hertel, 2012). The problem is that the standard CET approach uses a land frontier that reflects "productivity adjusted area" not "physical area" of land.

CGE models that use CET approach usually apply scaling methods to maintain physical area of land in balance[5]. More recently, several papers have used Extreme Value distribution functions to allocate land among uses and maintain the area of land in balance (Costinot et al., 2016; Fujimori et al., 2014; Gouel and Laborde, 2018; Sotelo, 2019). Furthermore, following the original work developed by Dixon and Rimmer (2003)[6], van der Mensbrugghe and Peters (2016) introduced a new approach, named Additive CET (ACET), to maintain physical area of land in balance while allowing land allocation to respond to changes in relative returns.

[1] Some authors used other approaches such as land transformation matrices (e.g., Ferreira Filho et al. (2015)), cost of land conversion (e.g., Gurgel et al. (2007)), and simple market clearing conditions (e.g., Sands et al. (2014a)).

[2] For the first time these authors defined and used a CET function to represent a production frontier with multiple agricultural products and take into account the opportunity costs of choices among these products.

[3] A few examples are: Hertel and Tsigas (1988), Darwin et al. (1995), OECD (2001), Banse et al. (2008), Eickhout et al. (2009), Hertel et al. (2010), Palatnik et al. (2011), Laborde and Valin (2012).

[4] As shown later in this paper, a model that considers land as a mobile input ignores these costs.

[5] For example, Golub and Hertel (2012) and Laborde (2011) have explained the scaling methods used in the GTAP-BIO and MIRAGE-BIOF models.

[6] These authors defined an additive form of constant elasticity of substitution (ACES) function for labor aggregation.

While these methods all successfully retain physical area of land in balance, their merits and implications for land use and economic analyses have not yet been fully examined. This paper compares these approaches and examines their merits, limitations and economic consequences. We develop analytical and numerical analyses to examine:

1) the extent to which the CET approach generates imbalances in land area;
2) what factors determine the magnitude of such imbalances;
3) how alternative approaches remove the imbalances;
4) and the limitations of these approaches.

2. Theoretical background

2.1. CET approach

The CET approach was developed based on an optimization problem in which the hypothetical landowner allocates a fixed amount of land across uses given a set of land rents in order to maximize total land rental revenue. The literature usually defines this optimization problem using the following set up:

$$\text{Max} \;\; \sum_j P_j X_j \tag{1}$$

$$\text{Subject to: } V = \left[\sum_j \alpha_j X_j^{-\rho}\right]^{-\frac{1}{\rho}} \tag{2}$$

In this optimization problem P_j is the rental price of land[7] type (or use) j, X_j indicates "physical area" of land type j, α_j and ρ are given parameters and $\rho \leq -1$. The function $V = \left[\sum_j \alpha_j X_j^{-\rho}\right]^{-\frac{1}{\rho}}$ represents a standard CET function[8]. Henceforth, we refer to this function as: $f(\cdot)$. For a given set of land prices, this optimization problem determines $X_j \; \forall j$ for a certain value of V (usually defined as "productivity adjusted land"[9]) and the technology of land conversion defined by the function: $f(\cdot)$. The land constraint of $V = f(\cdot)$ does not impose a restriction on total physical area of land. However, it represents feasible combinations of land uses from an economic point of view. While the CET approach does not impose a physical constraint on total area of land, CGE models typically assume that this restriction holds in the benchmark database. This means that at the initial equilibrium $X = \sum_j X_j$. Here, X represents total physical area of land.

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[7] In this paper price of land represents annual rent per unit of land.

[8] In practice CGE models may use multi-nests CET function. In addition, CGE models include some additional parameters to take care of changes in productivity. To make the notation simple, without losing generality, we did not include productivity parameters. Finally, in a CET function $\rho = \frac{1-\sigma}{\sigma}$, where $\sigma < 0$ represents transformation elasticity.

[9] For a simple interpretation of this concept see Gohin (2016).

Consider now a CGE model that uses the CET approach and traces percentage changes in land allocation in transition from an initial equilibrium to a new equilibrium (say due to a shock in demand for a land using sector) assuming that the land endowment (i.e., V) is fixed. For this model it is straight forward to show that[10]: $v = \sum_j \theta_j x_j$. Here θ_j represents the revenue share of land use j, and variables v and x_j show percentage changes in variables V and X_j. Since the land endowment is fixed, that implies:

$$v = \sum_j \theta_j x_j = 0. \tag{3}$$

This suggests that from an initial equilibrium to a new equilibrium, the economy can only move along the land frontier.

On the other hand, from the physical land constraint it is clear that:

$$x = \sum_j \Theta_j x_j, \tag{4}$$

where Θ_j represents the share of land type j in total land area and x represents percentage change in total area of land. Since land prices vary across uses, one can conclude that: $\theta_j \neq \Theta_j$ at least for some j. Therefore, from equations (3) and (4) it is straight forward to conclude that $x \neq 0$. This means that with the CET approach, total physical area of land cannot remain fixed in moving from one equilibrium to another one. In fact, $x \neq 0$ confirms an imbalance in physical area of land.

Figure 1 demonstrates a simple graphical analysis for a case with two types (or uses) of land: X_1 represents physical area of land type 1 and X_2 shows physical area of land type 2. In this figure the initial equilibrium is represented at point A with an allocation of total physical area of land between land types 1 and 2 demonstrated by X_1^A and X_2^A, respectively. At this point the slope of $f(\cdot)$ is equal to the relative land prices, $\frac{P_{X_1^A}}{P_{X_2^A}}$, which represents the slope of isorevenue at the initial equilibrium. The physical land constraint passes through point A as well. This means that the physical land constraint (i.e. $X = X_1^A + X_2^A$) holds at the initial equilibrium too. In this figure point B represents a new equilibrium where area of land type 1 has increased to X_1^B due to an increase in the price of this type of land. Figure 1 clearly shows that the new equilibrium (point B) is not on the physical land constraint defined by $X = X_1 + X_2$. However, it remains on the economic land frontier defined by $V = f(\cdot)$. All points on this curve satisfy $v = 0$ and all point on $X = X_1 + X_2$ satisfy $x = 0$. Since point B is not on the physical land constraint, then clearly $x \neq 0$. It is trivial that in this simple case:

$$X = (X_1^A + X_2^A) > (X_1^B + X_2^B). \tag{5}$$

This means a negative change in total physical area of land which implies $x < 0$. Note that the point B in Figure 1 is intentionally selected below the physical land

[10] See Appendix A for the derivation of this relationship.

constraint. The CET approach may select a point on the productivity adjusted land frontier which requires an expansion in physical area of land (i.e. $x > 0$), e.g., somewhere on the productivity adjusted land frontier to the right of point A where the land frontier goes beyond the physical land constraint.

The land supply function for the optimization problem defined above is derived in Appendix A and presented by equation A21. As shown by this equation basically two main items shape the supply functions derived from a CET approach: 1) the relative prices of land in alternative uses which reflect heterogeneity in land prices and 2) the land transformation elasticities. It is important to emphasize that these two factors jointly determine the size of imbalance in physical area of land in the CET approach as well. However, there is no clear analytical pathway to examine the extent to which these factors affect the size of imbalance. For this reason, later in this paper we accomplish this task by developing some numerical simulations.

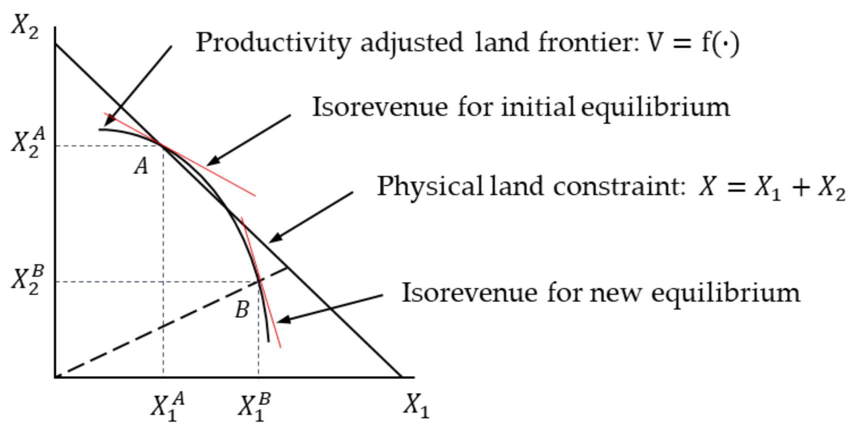

Figure 1. Land allocation in a simple CET approach

Notes: Point A represents an initial land allocation. This point is on both the CET land frontier and the physical land constraint. Point B represents a new land allocation induced by changes in the relative prices of land. This point is on the CET land frontier, but it is not on the physical land constraint. This implies imbalances in area of land at point B.

Source: Author's construction.

Several remedies have been proposed to fix the results of the standard CET approach and maintain physical area of land in balance. In what follows we outline these approaches and their implications. However, before that we should adjust the optimization problem defined by equations (1) and (2) to match it with the implemented set up that CGE models use in practice.

The CGE models operate based on the input-output tables that represent monetary values of sales and purchases, not physical quantities (van Tongeren et al., 2017). For instance, instead of area of land and their prices, the input-output tables reflect payments to this input across uses. In fact, these payments represent value added of land[11] by sector/use. For this reason, it is more accurate to rewrite the CET optimization problem using the monetary variables as described in the following:

$$\text{Max } \sum_j R_j Q_j \tag{6}$$

$$\text{Subject to: } Q \leq \left[\sum_j \alpha_j Q_j\right]^{-\frac{1}{\rho}}. \tag{7}$$

In this optimization problem Q_j indicates the payments to land in use j, evaluated at initial equilibrium prices, and R_j represents the relative price of land type j over the numeraire used in the CGE model. Hence, in these models Q_j does not measure physical area of land. One can consider Q_j as the proxy for the productivity adjusted land in use j. Henceforth, we refer to function $Q = \left[\sum_j \alpha_j Q_j^{-\rho}\right]^{-\frac{1}{\rho}}$ as: $g(.)$. This function represents land transformation technology and variable Q shows a given value of the overall "productivity adjusted land". For this optimization problem which measures land in monetary values (at fixed initial prices), it is straight forward to show that: $q = \sum_j \theta_j q_j$. As defined before, θ_j represents the revenue share of land use j and variables q and q_j show percentage changes in variables Q and Q_j. Following the reasons, we used for the first optimization problem, one can simply conclude that if $q = 0$ and $x_j = q_j$, then $x \neq 0$. As mentioned before, this suggests that the CET approach fails to keep area of land in balance.

Figure 2 represents our graphical analysis for the set up that has been used by most CGE models in practice. This figure represents two panels: the left shows monetary values (value added of land in each use) and the right represents physical area of land (area of land in each use).

The left panel of Figure 2 shows the initial equilibrium at point A. At this point the slopes of productivity adjusted land frontier (i.e., the slope of function $g(\cdot)$) and the initial isorevenue are identical, and both are equal to one. That is because at the initial equilibrium all prices are equal one in CGE models. Hence at this point $P_{X_1^A} = P_{X_1^A} = 1$ and therefore: $\frac{P_{X_1^A}}{P_{X_2^A}}$ =1. The allocation of physical area of land corresponding to the initial equilibrium is presented at point A on the right coordinate. In this coordinate, point A is on the line defined by: $X = X_1 + X_2$. In the presence of heterogeneity in land prices, the revenue shares and their corresponding land shares are not identical at the initial equilibrium. For this

[11] In an input-output table, payments to the primary inputs such as land, labor, capital and resource represent value added of those inputs. In the Global Trade Analysis Project (GTAP) Data Bases, EVFA and VFM represents value added of these inputs at agent and market prices.

reason, the initial equilibrium on the monetary and physical coordinates are intentionally selected to represent a case where $\theta_1 < \Theta_1$. In moving from the initial equilibrium to the new equilibrium, the economy moves on the productivity adjusted land frontier from point A to point B on the monetary (left hand) panel. However, on the right panel which represents physical area of land, point B falls below the physical land constraint, when we assume $x_j = q_j$. Under this assumption given that $\theta_j \neq \Theta_j$ then: $x \neq 0$. This reflects an imbalance in land area.

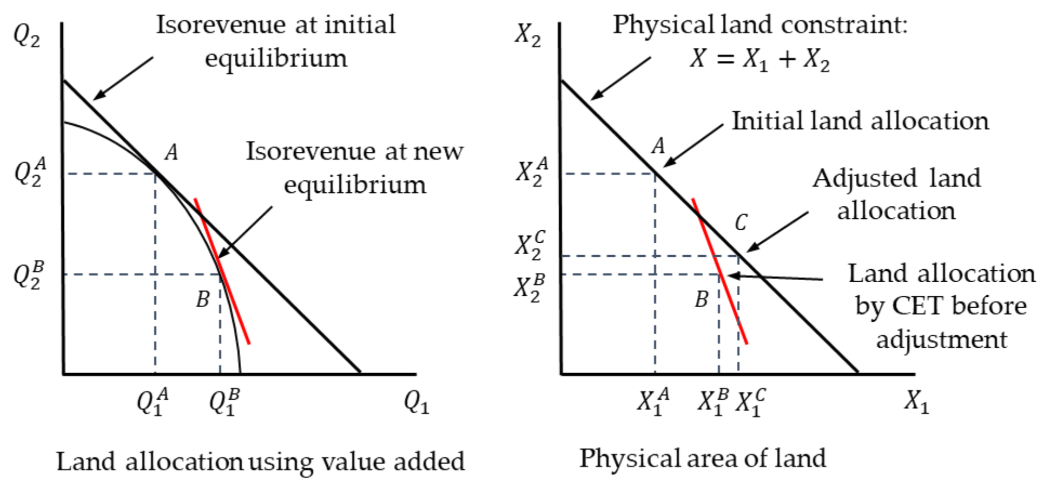

Figure 2. Land allocation using monetary values of land in a typical CGE model

Notes: The left panel shows a CET land frontier defined on the monetary values of land (value added) and the right panel represents the physical land constraint. On each panel, point A shows an initial equilibrium and point B indicates a new allocation induced by changes in land prices. The right panel shows that there are some imbalances in land area at point B, because it is not on the land constraint. Point C shows land allocations after adjustments that remove imbalances.

Source: Author's construction.

Several scaling approaches have been used to restore physical land in balance. In fact, these approaches define some arbitrary mapping to transfer the CET results to a new point on the physical land constraint, say transfer point B on the physical panel of Figure 2 to point C in this panel. In what follows we examine these alternative mapping approaches.

2.2 Removing imbalance in physical area of land generated by CET approach

To maintain physical area of land in balance one can use either *ex post* or *ex ante* scaling approaches. The *ex post* approaches convert a given solution obtained from the CET solution to physical area of land. The *ex-ante* approaches actively make the adjustments during the simulation process.

2.2.1 *Ex post* scaling

For the *ex post* scaling approach, we introduce two methods. The first method assumes $x_j = q_j + s$ and $\sum_j \Theta_j x_j = 0$, where s is an endogenous slack variable and q_j is predetermined by the CET results. In this approach the signs of x_j and q_j may not match. This means that the percent changes in physical land and productivity adjusted land may have different signs in the same use. While this seems odd, it could be because of changes in average productivity of a particular type of land use due to entry and exit of land from and to that type of land use[12]. The alternative versions of GTAP-BIO model, regardless of details, basically use this approach (Golub and Hertel, 2012; Hertel et al., 2010; Taheripour et al., 2017).

The second *ex post* scaling method, proposed by Horridge (2014), assumes $x_j = \alpha_j q_j$, where $\alpha_j = \frac{(Q_j/\sum_j Q_j)}{(X_j/\sum_j X_j)}$. In fact, this method scales the CET results by a factor that measures disparity between the land revenue and land area shares which implicitly takes into account heterogeneity in land prices.

In general, the *ex post* scaling methods restore the balance in physical area of land in an *ad hoc* manner. However, they do not alter the CET results. In particular, in these approaches, the welfare calculation remains consistent with the classical welfare analysis provided by the CGE model. Figure 2 represents an *ex post* scaling approach which only transfers B to C on the physical coordinate with no change on the monetary panel.

2.2.2 *Ex ante* scaling

Unlike the *ex post* scaling methods, the *ex ante* methods essentially alter the CET approach to maintain physical area of land in balance. Hence, these methods alter the CET results and generate some unusual welfare consequences. For the *ex ante* approach, we introduce two methods.

As mentioned before, the CET approach adopts the assumption that productivity adjusted area of land is fixed (i.e. $q = 0$). The first *ex ante* scaling method alters this assumption and assumes: $x = 0$. To adopt this assumption, one needs to add x to the list exogenous variables and swap that with q, which is an exogenous variable in the original CET approach. With this swap q becomes an endogenous variable and can change. In addition, the following two equations should be added to the model: $\sum_j \Theta_j x_j = x$ and $x_j = q_j$. In fact, this method shifts the land frontier to maintain $x = 0$. In other words, this method scales up/down the CET land frontier to meet the physical land constraint, while it maintains the original curvature of this function. This causes some changes in the productivity

[12] For example, while area of wheat goes up, productivity adjusted area of wheat may fall. That could happen when some productive lands (say 10 hectares) moved from wheat production to other applications and some low productivity lands (say 15 hectares) move to wheat production.

adjusted land endowment which generates some unusual welfare impacts compared with the conventional CET approach. Henceforth we refer to this approach as Modified CET (MCET).

Figure 3 represents this method of adjustment. The monetary coordinate of this figure shows a shift in the land frontier induced by fixing total area of land and allowing changes in V. While figure 3 represents a positive shift in the land frontier, in principle, the shift could be negative as well. So, while the original CET approach projects point B as the new equilibrium the MCET version selects point C where the economy gains from having an increase in the productivity adjusted land. In reality, moving from point A to point B would entail some opportunity costs. However, in moving from point A to point C in this approach, the economy gains and pays nothing for the transition.

The right panel of Figure 3 represents land allocation in physical area for points A, B, and C. This figure shows that points B and C fall on the line passing through the origin of the physical land coordinate. This indicates that point C is a linear transformation of point B. Our numerical work presented in the next section confirms this property.

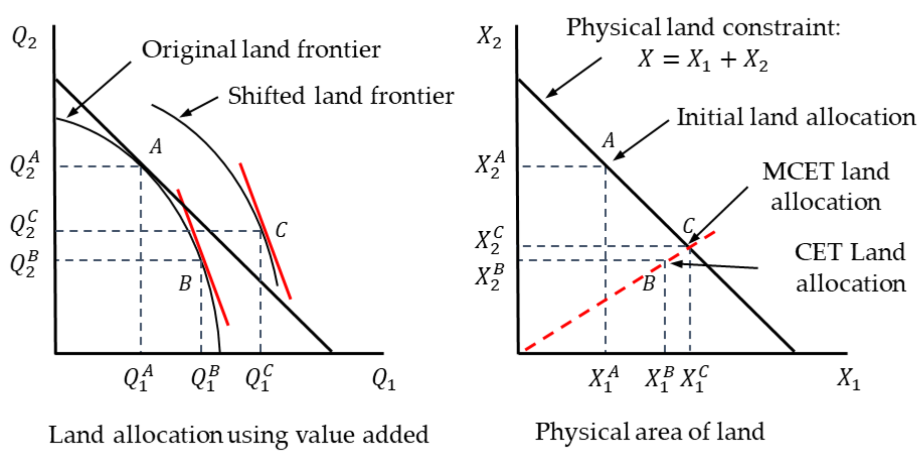

Figure 3. Land allocation using MCET method

Notes: The left panel shows a CET land frontier defined on the monetary values of land (value added), an initial equilibrium (point A), a new equilibrium (point B) induced by changes in the land prices, a shift in the land frontier to remove imbalances in land area, and the new equilibrium after land adjustment (point C). The right panel shows the physical land constraint.

Source: Author's construction.

For the second *ex ante* method, we refer to the Additive CET (ACET) approach introduced by van der Mensbrugghe and Peters (2016)[13]. This approach alters the CET optimization in the following form[14]:

$$\text{Max} \quad U = \left[\sum_j \alpha_j \left(P_j X_j\right)^{-\rho}\right]^{-\frac{1}{\rho}} \tag{8}$$

$$\text{Subject to:} \quad \mathrm{X} = \sum_j X_j \tag{9}$$

Unlike the standard CET approach which maximizes revenue for a given land frontier, the ACET maximizes U (a function of revenues) subject to a physical land constraint. Appendix A shows the land supply function obtained from this optimization problem. As explained in the appendix, one cannot easily provide a mapping relationship between the land supply functions of CET and ACET with level variables. However, Zhao et al. (2020b) have shown that in percent change form an ACET land supply function can be decomposed into a shifting factor and a CET land supply function (for details see section 3 and 4 of Appendix A). This means that in percent change ACET shifts the CET frontier to meet the physical land constraint, which follows the same intuition with the MCET approach discussed above. In fact, as will be shown below, both ACET and MCET approaches provide the same numerical results. Hence, similar to the MCET, the ACET approach generates some unusual welfare consequences.

In summary, from the analyses provided in this section we reach the following conclusions: 1) the standard CET fails to maintain physical area of land in balance; 2) heterogeneity in land prices and the curvature of the land frontier affect the size of imbalance; 3) the *ex post* scaling approaches, which respect conventional economic analyses restore area of land in balance, but their results are subject to personal judgment; and 4) *ex ante* scaling approaches maintain area of land in balance with some unusual welfare implications.

Section 2.3. Land allocation using extreme value distribution functions

An alternative approach in the literature uses extreme value distribution functions to model land allocation across uses. In this section, we first explain properties of this approach using a simple stylized model. We then review the most recent papers that used this approach. We finally use three stylized CGE models that represent the same demand side but different land allocation approaches - including CET, MCET, and an extreme value distribution function (Fréchet) - to compare their outcomes. We highlight similarities and differences across these models regarding their implications for welfare and land use.

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[13] Horridge (2014) also defined a method similar to the ACET approach.

[14] One can write this optimization problem using the monetary values as we did for the standard CET.

2.3.1. Model description

In this approach, the available land consists of a continuum of plots and land productivity for every crop which are heterogeneous across these plots. Agricultural producers decide which crop to grow in every plot[15]. To explain properties of this class of models clearly, we consider a stylized framework in which land is the only factor of production[16], and the distribution of disaggregated land productivities is Fréchet (extreme value type II). We summarize the main takeaways here and refer the reader to Appendix B for a full description of this model. The Fréchet distribution specification allows for the aggregation of discrete choices into a closed-form equation. Two equations are crucial to the behavior of this model. First, the share of land allocated to crop j is given by Π_j:

$$\Pi_j = \frac{X_j}{X} = \frac{(P_j a_j)^{\varphi}}{\sum_{j\in J}(P_j a_j)^{\varphi}} \tag{10}$$

Where P_j is price of crop j and J is the set of crops. In this equation $\varphi > 1$ represents the dispersion parameter of the distribution function. Parameter a_j is a productivity shifter for crop j and determines the average disaggregated productivities for crop j across the entire available land. By structure, land shares sum up to one, and so, the model necessarily preserves the balance of total land. Second, the supply quantity of crop j is given by

$$Q_j = X a_j \Pi_j^{(\varphi-1)/\varphi} \tag{11}$$

We emphasize that the production functions are constant returns to scale (homogenous of degree 1) at the level of individual producers whereas they are homogenous of degree $\frac{\varphi-1}{\varphi} < 1$ at the aggregate level. This implies that the aggregate land productivity (i.e. yield) is itself endogenous. We can decompose the channels through which crop supply changes in response to shocks or policy:

$$\underbrace{\Delta \ln Q_j}_{\%\ cha.\ in\ supply} = \underbrace{\Delta \ln(X_j)}_{\%\ cha.\ in\ land\ use} + \underbrace{\Delta \ln(Land\ Productivity)}_{\%\ cha.\ in\ productivity}$$

$$= \underbrace{\Delta \ln(\Pi_j)}_{\%\ cha.\ in\ land\ share} + \underbrace{\Delta \ln(a_j)}_{\%\ cha.\ in\ exog.\ productivity} + \underbrace{(-1/\varphi)\Delta \ln(\Pi_j)}_{\%\ cha.\ in\ endog.\ productivity} \tag{12}$$

Showing % changes in level variables and also percentage change in Π_j with their corresponding small letters and denoting $\dot{a} = \Delta \ln a$, we rewrite the decomposition as:

[15] One can extend this approach to the land allocation among crops, pasture, or forest as well.

[16] The production function can be extended for multiple inputs as well.

$$q_j = \pi_j + \dot{a}_j + (-1/\varphi)\pi_j \tag{13}$$

Everything being equal, the aggregate yield decreases with the land share. Underlying to this effect is that: 1) land is heterogeneous and 2) as land use of crop j expands, less suitable parts of the available land will be used for crop j. Hence, the average yield falls for crop j. Quantitatively, the Fréchet-based structure imposes a constant elasticity on this relationship, $(-1/\varphi)$. In addition, φ has another interpretation within the model as it is also the elasticity of land use with respect to the crop price.

One advantage of this model is that it provides structural relationships between yields, land share, and prices, which could be used to estimate φ. As a limitation, this model imposes that land rents must be equalized across uses.

2.3.2. Review of the related literature

Costinot et al. (2016) implemented a Fréchet-based model of land allocation using detailed global grid-level data from FAO-GAEZ. These data measure the potential yield as the yield of a grid cell if the entire grid cell was allocated to a crop. The model tightly maps to these data since the model implies:

$$\left[\frac{Q_j}{X_j} = a_j \mid \Pi_j = 1\right] \tag{14}$$

Therefore, the shifter a_j for every grid cell is precisely what FAO-GAEZ reports as the potential yield in that grid cell. Note that because yield is endogenous in the model, the predicted yield will be different from the potential yield (unless the entire land is allocated to a crop). Costinot et al. (2016) solve their model with the potential yields of the contemporary baseline and compare it with their model prediction when potential yields are based on different scenarios of climate changes.

Sotelo (2019) generalizes a similar structure to include other factors of production in addition to land. He uses detailed data for both potential yields as well as prices of crops. Exploiting variations in potential yields, prices, and land shares he estimates the elasticity φ. A simplified version of his estimation is as follows. He employs the land share equation:

$$ln(P_j a_j) = \left(\frac{1}{\varphi}\right) ln\, \Pi_j + \cdots \tag{15}$$

Wherea_j is the potential yield as in the case of Costinot et al. (2016), and P_j and Π_j are observed. The unit of observation is a pairing of grid cell and crop. The dots in equation (15) represent mismeasurements in data as well as fixed effects for grid cells and crops. In Sotelo, the main experiment involves changing domestic trade costs between regions within a domestic economy (e.g., an infrastructure project).

These models have the potential to be extended for a variety of applications. For example, Gouel and Laborde (2018) add a livestock industry to the model of

Costinot et al. (2016). In this specification, the non-cropland is used in the livestock production rather than remaining unproductive. Farrokhi and Pellegrina (2019) extend the Fréchet-based approach to incorporate choices of technologies in addition to crops. This specification introduces a new elasticity that governs the extent to which agricultural producers adopt input-intensive technologies for every crop as agricultural input prices change.

Closely related to the Fréchet-based approach is the logit approach (Fujimori et al., 2014). An early application of the logit formulation in land use models was developed in Sands and Leimbach (2003) based on a log transformation of the Gumbel distribution (extreme value type I), which is identical to the Fréchet distribution (extreme value type II) and thus resulted in the same land sharing formulas shown above in Section 2.3.1 However, the empirical application of the approach, e.g., in Fujimori et al., 2014 and Wise et al. 2014, has been simplified to a land supply function that is mathematically identical to the ACET approach to permit differential rental prices across land uses (Zhao et al., 2020a).

2.3.3. Lessons from a numerical exercise

In order to shed light on similarities and differences between models of land use based on Fréchet, CET, and MCET, we consider a stylized CGE model which is presented in full length in Appendix C. We let land be the only factor of production, specify utility with a CES function, and consider three specifications on the production side based on Fréchet, CET, and MCET. We then show how to calibrate these three models to any base data on crop land uses and crop output quantities. Having the three models reproduce the exact prices and quantities in the baseline, we consider a counterfactual subsidy on the consumption of corn. Then we evaluate the predictions of each of these three models in response to the policy.

We refer the reader to Appendix C for detailed equations and results, and here summarize the main lessons from our exercise. We put our summary into two comments.

First, we compare Fréchet with CET[17]. Recasting the land allocation problem to crop supply allocation, we show that Fréchet and CET imply the same curvature of production possibility frontier across crop quantities if $\rho = \varphi/(\varphi - 1)$. Because total efficiency of land, V, is an exogenous variable in CET, the production possibility frontier does not shift. These features cause Fréchet and CET to

[17] One can define the exponent of a CET function as $-\frac{1}{\rho}$ when $\rho < -1$. An alternative option is to use $\frac{1}{\rho}$ for the exponent when $\rho > 1$. Unlike this section and appendix B, in this paper we use the first definition to be consistent with the notation used in the GTAP literature and code. However, in this section and Appendix B we follow the second definition to make the comparison between the CET and Fréchet functions more straightforward.

generate exactly the same changes to prices and quantities of crops, hence the same changes to welfare.

However, differences between Fréchet and CET do show up in their predictions for changes in land use, land rents, and yields. In our exercise, since subsidies distort market prices, welfare has to decrease. As explained above, the reduction in welfare will be exactly the same between Fréchet and CET. A lower welfare as a result of exactly the same changes to crop quantities is implied in two different ways between Fréchet and CET. Fréchet induces yield changes, whereas total physical land remains exogenously unchanged. In contrast, CET implies a reduction in total physical land whereas yields remain exogenously unchanged.

Second, Fréchet and MCET (which is identical to ACET) necessarily have differing implications for welfare. The underlying reason is that in response to price changes, the location of production possibility frontiers (both for land use and crop output) involve a shift in the case of MCET. In our exercise the reduction in welfare is almost three times smaller in the case of MCET compared to Fréchet or CET. In our stylized model, we have:

$$\Delta \ln U^{MCET} - \Delta \ln U^{CET} = \Delta \ln V^{MCET} = -\Delta \ln X^{CET} \quad (16)$$

Or in percent change notation:

$$u^{MCET} - u^{CET} = v^{MCET} = x^{CET} \quad (17)$$

Where U is welfare (utility level), and V is total efficiency units of land. The difference between the change in utility in MCET compared to CET is the compensation through the increase in total efficiency units of land V in MCET, which exactly equals the reduction to total physical land X in CET.

The takeaway from our discussion is that, as long as welfare is of concern, then Fréchet and CET generate equivalent predictions – provided that $\rho = \varphi/(\varphi - 1)$, and that they are calibrated to the same base data on crop quantities. However, if land-related variables such as rents, yields, area of land, or land use emissions are of concern, then the two approaches provide different predictions. In comparison, MCET have different predictions compared to Fréchet and CET for both land use and welfare. Similar to what we found before, MCET preserves the land shares generated by CET, while it maintains total area of land in balance.

Fréchet has no advantage over CET (neither CET over Fréchet) as long as the research goal is to study the output and prices of crops and the welfare from crops consumption. However, if the research goal is to study land use change, then this approach has an advantage over CET. While Fréchet has this advantage, its implementation has two requirements: 1) it requires equal land rents across uses in the benchmark data and 2) the calibration process must take into account yields across uses. The implementation of these requirements in a typical CGE model employing the GTAP Data Base is not trivial. The GTAP Data Base shows heterogeneous land rent across uses, in particular across land cover items including cropland, pasture, and forest. Even land rents across crops are not identical in the GTAP Data Base. Hence, reconciling the land rent implications of Fréchet with GTAP base data requires nontrivial extensions, which we leave for future work.

We can certainly devise land-use allocation mechanisms that (a) are profit-maximizing to the landowner (so avoiding strange welfare effects); (b) are consistent with an initial disparity in per-area land rents; and (c) conserve total land area. Horridge (2019) gives an example, based on the CRETH function. But so far no such function has been used in a CGE model and calibration remains a challenge.

2.4. Land allocation with explicit costs of land conversion

As mentioned before, the CET approach implicitly takes into account the opportunity costs of land conversion. From a landowner perspective[18], these opportunity costs can be divided into two groups. The first group covers the losses in value added of land (land rent) in its current use. The second group is the cost of land conversion due to the curvature of the CET land frontier. The land transformation elasticities impose the second group of opportunity costs. The larger the size of land transformation elasticity the smaller the opportunity costs. A linear CET frontier which basically reflects a simple market clearing condition undermines the second group of opportunity costs and only takes into account the first group.

The CGE models that use stochastic distribution functions take into account the losses in land rent in current uses when land is moved from one application to another one. They also take into account the second group of land conversion costs that the CET approach imposes on the land allocation process through the dispersion parameter of the distribution function (i.e., φ). However, the CET and stochastic distribution approaches both ignore the explicit preparation costs of moving land from one application to another one.

Gurgel et al. (2016) reported a set of unique modeling efforts that explicitly introduce costs of land conversion into a global CGE model. These efforts were made to explicitly introduce land conversion and preparation costs into the MIT Economic Projection and Policy Analysis (EPPA) model, maintain physical area of land in balance, and find an optimal allocation of land across uses subject to economic and biophysical constraints. To accomplish these goals land conversion activities were defined and included in this model. These activities use primary inputs (capital and labor) and intermediate inputs (including energy) to convert land from one application to another application. Given the land conversion costs, then a mixed complementary optimization problem was used to allocate land

[18] In a CGE model any change in the land allocation generates a series of changes in other markets that generate some welfare impacts.

across uses. This optimization problem considers land allocation as a cost-benefit analysis that compares the land conversion costs with land rent benefits of the conversion to find an optimal land allocation. The EPPA model assumes that the converted land from use k to use g takes the average productivity of land in use g.

It is not clear, who pays the costs of land conversion and how these costs are distributed over time. In addition, it is not clear to what extent the estimated costs match real observations on land conversion costs.

3. Numerical Analyses for CET based approaches

3.1 Model and examined experiments

To support our theoretical analyses, we developed several numerical simulations using the GTAP-BIO model used by Hertel et al. (2010). The model used the first version GTAP-BIO Data Base developed by Taheripour et al. (2008). These authors introduced biofuels and their by-products into the standard GTAP Data Base version 6 (McDougall and Dimaranan, 2002). This model divides the land endowment of each region by up to 18 Agro-Ecological Zones (AEZs). There is no land movement across AEZs. The land endowment of each AEZ is divided across the land using sectors including forestry, livestock sectors (ruminant and dairy) and several crop sectors. The land supply side of this model uses CET functions to allocate land across the land using sectors. Then it uses one of the *ex post* scaling methods that we introduced above to maintain physical land area in balance. The model operates based on monetary values to move from an initial equilibrium to a new equilibrium. Then it uses the simulation results and data on physical area of land by AEZ to calculate changes in physical area of land. The model uses a two-stage nested CET functional form. To concentrate on the subject under investigation, we changed the model to represent a one-nest CET. We use the revised model and its alternatives to examine: the extent to which the standard CET generates imbalances in land area; welfare implications of alternative types of scaling methods; and relationship between standard CET, MCET, and ACET approaches.

We examined all of these cases for an exogenous increase in US corn ethanol production/consumption from 1.68 billion gallons to 14.24 billion gallons, an expansion by 747% or 12.56 billion gallons. The GTAP-BIO model uses a revenue neutral tax-subsidy mechanism to encourage consumption of ethanol versus traditional fuels. It basically taxes fossil fuels and subsidizes ethanol consumption to increases consumption of ethanol to the desired level.

The implemented shock in US corn ethanol is a large shock and significantly alters crop demands and induces relatively major land use changes in the US. It has implications for other regions as well. The simulation results that we provide in the following section reflect the impacts of this particular shock under alternative cases. However, the conclusion that we make from the results are independent of the implemented shock.

3.1.1 Size of imbalance in land area with standard CET

To understand the extent to which the standard CET generates imbalances in land area and determine its sources, we examined three simulations with three different land transformation values. The first simulation examines the size of imbalance by region and AEZ with a relatively small size of land transformation elasticity of $\sigma = -0.5$ used by Hertel et al. (2010). This rate of land transformation represents a relatively small transformation rate in land use. The second simulation repeats the first one with a large land transformation elasticity of $\sigma = -10$. This simulation relaxes the rate of transformation among land uses and represents a relatively flat CET frontier. Note that the exponent of the CET frontier ($\frac{-1}{\rho}$) varies between 0 and 1, where 0 shows no transformation among land uses and 1 represents perfect substitution. Hence, when $\sigma = -0.5$ then $\frac{-1}{\rho} = 0.33$, which represents a limited degree of transformation among land uses. However, when $\sigma = -10$ then $\frac{-1}{\rho} = 0.91$, which represents significantly a larger transformation rate among land sues. Clearly the second option generates a flatter land frontier compared to the first one.

In the third simulation, we replaced the CET frontier with a simple market clearing condition for land in all regions and AEZs. In fact, the third case is an extreme case of CET when the land transformation elasticity tends to infinity and represents perfect transformation among land uses.

The simple market clearing condition does not require to begin with equal land rent across uses in a given AEZ of a country. It imposes no restriction on land allocation on the supply side of the market for land and considers land as a mobile input, not a sluggish one. In this case price of land (rent per hectare) changes at the same rate across uses in each AEZ/country. This does not mean equal land prices across uses. With a simple market clearing condition for land, in a given country and AEZ, the price of land will change at the same rate across uses in the simulation process. That means: $p_i = p_j$ for $i \neq j$, where p_j represents the percent change in the price of land in use *j*. But this does not mean that the price of land will be equal across uses after simulation. The price of land in use *j* after simulation (P_j^a) will be equal to: $P_j^a = P_j^0(1 + p_j/100)$. Here, P_j^0 represents the price of land in use *j* before simulation. So if $P_j^0 = P_i^0$, since $p_i = p_j$, then $P_j^a = P_i^a$. Otherwise, if $P_j^0 \neq P_i^0$, since $p_i = p_j$, then $P_j^a \neq P_i^a$. Finally, we note that the changes in the price of land could be different across AEZs or countries.

In these three simulations, we turn off the *ex post* scaling embedded in the model to only work with the results with no adjustment. We refer to these simulations as simulations CET1, CET2, MARKET.

3.1.2 Welfare implications of MCET

As mentioned before, the MCET method uses a physical land constraint instead of the conventional land frontier to deal with imbalances in physical area of land. To examine the welfare implications of this alternation, we modify the GTAP-BIO model to represent this approach as described earlier in section 2.2.2. In this simulation we used the land transformation elasticity of $\sigma = -0.5$ to make its results compatible with CET1 simulation.

3.1.3 Welfare implications of ACET

As mentioned before, the ACET method basically follows the logic behind the MCET method. To confirm this fact we modify the GTAP-BIO model to represent the ACET approach as described by van der Mensbrugghe and Peters (2016). To compare the results of this simulation with the results of CET1 and MCET we used $\sigma = -0.5$.

3.1.4 Welfare impacts of *ex post* scaling methods

Since the *ex post* scaling methods remove imbalances with no welfare impacts, we did not examine the welfare impacts these methods.

3.1.5 Land allocation under alternative choices

The existing literature confirms that land allocation approaches (either CET or stochastic approaches) provide similar patterns in land allocation, regardless of the imbalance issue. This is an important finding and is a key for future research on land allocation in CGE models. To better understand the role of land allocation approaches, we developed a new experiment base on the MARKET case that was introduced above with an important difference. The MARKET case uses a value share market clearing condition[19]. The new case operates based on an area share to clear the market[20], while it takes care of the endowment effects of moving land on the physical land constraint. From this point of view, the new experiment is similar to MCET. But instead of CET, it uses a simple market clearing condition. We refer to this experiment as Physical Area Market Clearing (PAMC) allocation.

[19] For the MARKET case, the land market clearing condition is a revenue share equation: $\sum_j \theta_j x_j = 0$, where θ_j represents revenue share land type j in total land revenue.
[20] For this case, the land market clearing condition operates based on a physical land share equation: $\sum_j \Theta_j x_j = 0$, where Θ_j shows the share of land type j in total land area.

4. Simulation results

4.1 Size of imbalance in land area: Results of CET1, CET2, and MARKET

We calculated the size of imbalance in land area measured as percentage change over total area of land by AEZ and region[21] for these three simulations as presented in Appendix D. Table 1 highlights the results for US. This table indicates that the size of imbalance varies across AEZs regardless of the rate of land transformation. In particular, the rate of imbalance is relatively large in AEZ10 and AEZ11. More corn (the feedstock in demand for corn ethanol) is produced in these two AEZs relative to others AEZs. The shock in production of corn ethanol generates more land movement across uses in these two AEZs, and that generates larger imbalances.

Table 1 shows that the sign of imbalance could be positive or negative. This confirms that the CET approach (even a simple market clearing condition) may lose or gain area of land. In addition, this table indicates that in each AEZ the size of imbalance varies slightly with changes in the size of land transformation rate. In each AEZ, the size of imbalance is slightly lower with the simple market clearing condition. These results confirm that the curvature of CET is not a major determinant for the size of imbalance.

Table 1. Percent imbalances in US land area for CET1, CET2, and MARKET experiments

AEZ	CET1: $\sigma = -0.5$	CET2: $\sigma = -10$	MARKET
AEZ7	-2.00	-1.62	-1.54
AEZ8	-1.96	-1.34	-1.24
AEZ9	-2.36	-1.87	-1.79
AEZ10	-4.54	-4.60	-4.55
AEZ11	-4.32	-4.26	-4.20
AEZ12	-0.88	-0.52	-0.47
AEZ13	0.34	2.12	2.38
AEZ14	-0.45	0.75	0.93
AEZ15	0.18	1.37	1.53
AEZ16	0.02	0.05	0.06

Notes: In this table, percent imbalance for each AEZ under each experiment equals: $\frac{(Area\ after\ simualtion - Area\ before\ simualtion)}{Area\ before\ simualtion} \times 100.$

Source: *Author calculation`s.*

[21] The imbalance in each AEZ is determined by:
Imbalace $= \frac{(Area\ after\ simualtion - Area\ before\ simualtion)}{Area\ before\ simualtion} \times 100.$

When we change the size of land transformation elasticity from $\sigma = -0.5$ to $\sigma = -10$ (or when we use simple market clearing condition), the size of imbalance goes down (in absolute terms) slightly in most AEZs. In general, this confirms that a flatter CET generates smaller imbalances. However, one can see a change in the sign of imbalance in some AEZs when we alter the size of land transformation elasticity.

While the size of imbalance is not very sensitive to the curvature of the CET function, it is very sensitive to the heterogeneity of land rent among uses. To examine the extent to which heterogeneity in land rent among uses affect the size of imbalance we concentrate on the US AEZ10 where land rent varies largely across uses, as shown in Figure 4. In this AEZ the average rent is about $177 per hectare while the rents for forest and pasture are about $27, or 15% of the average rent. Table 1 shows an imbalance of 4.6% for this AEZ with the transformation elasticity of -10. We increased the rents for pasture and forest to represent 50%, 75% and 100% of the average rent and repeated the simulation with the same land transformation elasticity. The results show that the size of imbalance drops significantly from -4.6% to -1.44%, -0.67%, and -0.12%, respectively for the higher rents for forest and pasture. The heterogeneity in land rent among crops and the curvature of CET are responsible for the remaining imbalance (-0.12%) when the land rents for forest and pasture are equal to the average land rent.

Finally, it is important to emphasize two other points. First, the size of imbalance is relatively large only in US. In other regions the size of imbalance is small in general, as shown in Appendix D. More land use change occurs in the US compared with other regions, as the shock mostly affect the US land use. Second, the size of imbalance could grow with the size of shock in corn ethanol induced by ethanol production. The larger the shock in ethanol production, the larger the shock in demand for corn, and the larger the size of imbalance. A larger shock induces a larger movement on the land frontier which generates more imbalance. To examine the extent to which the size of imbalance responds to the shock in ethanol production we made two additional simulations. These simulations represent $\pm$ 25% deviation in the expansion of corn ethanol examined in the base case. The $\pm$ 25% deviation in the expansion in demand for ethanol is about $\pm$ 30% of the shock size applied in the simulation. We examined this test for the case of CET2. The results for the US AEZs are presented in Table D4 of Appendix D. This table shows that the size of imbalance changes in a symmetric way by $\pm$ 30% in each AEZ. The means that the size of imbalance changes with deviation in the shock size in a linear way.

In general, these results affirm that the curvature of the CET has some minor impacts on the size of imbalance. These results confirm that the heterogeneity in land prices is the main source of imbalance. The size of imbalance varies by region and AEZ. The major discrepancies occur in the areas where the land allocation changes in response to the change in demand for land. In our cases it occurred in

US where corn demand increased due to the implemented shock in demand for US corn ethanol.

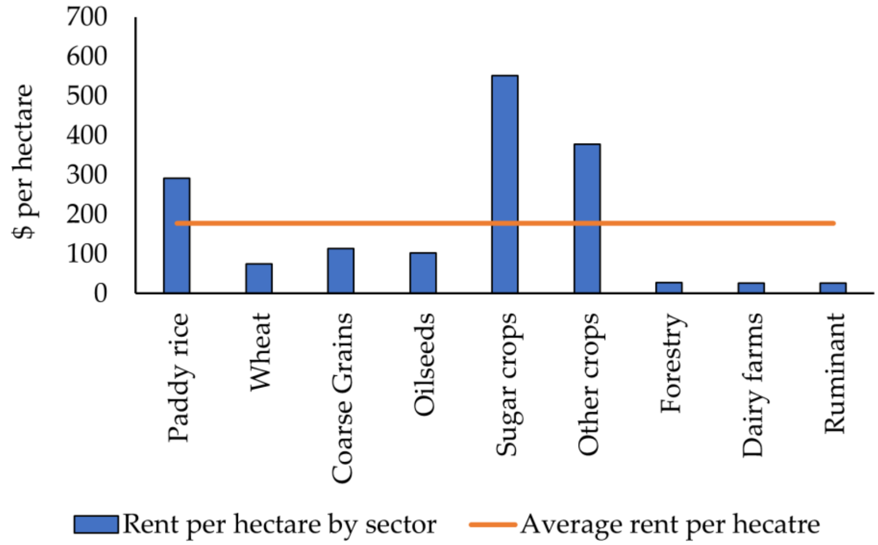

Figure 4. Rental rate of land in US AEZ10 by sector in 2011.

Source: GTAP-BIO Data Base.

4.2 Land allocation provided by alternative approaches

Unlike the CET1 results which represent imbalances in land area, the results obtained from MCET, ACET, and PAMC approaches show no imbalance in land area. We verify this in Table 2 which shows land allocation by sector for the US economy. The first columns of this table show allocation of land in the benchmark data base for US. The next three columns show projected land uses obtained from CET1, MCET, ACET, and PAMC simulations. The first important observation is that the total area of land for CET1 is less than the initial area of land by about 2.4%. This confirms the CET imbalance issue. The other three approaches show zero imbalances.

Table 2. Land allocation from examined experiments for US economy

Land use	Area of land (million hectares)				Land shares (%)			
	Initial data	CET1	MCET or ACET	PAMC	Initial data	CET1	MCET or ACET	PAMC
Paddy rice	1.3	1.3	1.3	1.3	0.2	0.2	0.2	0.2
Wheat	22.6	21.1	22.1	21.2	3.8	3.7	3.7	3.6
Coarse Grains	36.3	42.1	43.1	45.8	6.2	7.3	7.3	7.8
Oilseeds	32.6	30.7	31.9	31.6	5.5	5.3	5.4	5.4
Sugar crops	1.0	1.0	1.0	1.0	0.2	0.2	0.2	0.2
Other crops	38.3	36.5	37.7	37.4	6.5	6.3	6.4	6.4
Forestry	225.4	218.2	222.9	222.3	38.3	38.0	37.8	37.7
Dairy farms	99.0	96.0	98.0	97.8	16.8	16.7	16.6	16.6
Ruminant	132.8	128.1	131.2	130.8	22.5	22.3	22.3	22.2
Total	589.2	574.8	589.2	589.2	100.0	100.0	100.0	100.0
Imbalance%	0	-2.4	0	0	-	-	-	-

Source: Author calculations.

Regardless of the imbalance issue, one can also make the following inferences from Table 1:

The MCET and ACET generate the same outcomes (in area terms) for land use by sector, and those are different from the CET1 simulation results. The case of PAMC generates the largest expansion in coarse grains and more conversion of forest and pasture to cropland. That is because the simple market clearing condition that we used in this approach makes land conversion easier compared to other cases.

The CET1, MCET and ACET simulations generate the same land distributions across uses in terms of sectoral shares as shown in Table 2. That simply conforms that the MCET and ACET linearly transfer the CET results to a point on the physical area of land constraint. Notice that, here we showed the results aggregated over AEZs for US. This property also holds at the AEZ level in all regions worldwide.

Finally, for the case of PAMC, which imposes no restriction on land allocation and simply aggregates the demand side for land, the results in terms of land shares are only slightly different from the results of CET1, MCET, and ACET.

4.3 Crop outputs and prices provided by alternative approaches

Table 3 represents percent changes in US agricultural and forestry products and prices obtained from the examined cases. As shown in this table the results for MCET and ACET are identical but different from the CET1 simulation results. One can see the same pattern for all other variables of the model as well.

Table 3. Changes in US agricultural and forestry products and prices under alternative examined experiments for US economy

Sector	%Changes in supplies			%Changes in producer's prices		
	CET1	MCET or ACET	PAMC	CET1	MCET or ACET	PAMC
Paddy rice	-5.2	-1.8	-3.2	3.8	1.1	1.6
Wheat	-6.3	-2.2	-5.9	2.7	0.7	1.6
Coarse Grains	18.3	20.6	26.2	13.8	9.2	1.4
Oilseeds	-4.8	-1.7	-2.5	4.4	1.3	1.6
Sugar crops	-0.3	-0.2	-0.2	6.4	1.9	1.9
Other crops	-3.7	-1.3	-1.9	4.5	1.3	1.7
Forestry	-2.3	-0.9	-1.1	6.9	2.0	2.4
Dairy farms	-0.7	-0.3	-0.3	2.7	1.0	0.7
Ruminant	-1.3	-0.6	-0.6	2.6	0.8	0.7

Source: Author calculations.

One important observation is that the PAMC simulation projects more supply of coarse grains and more reduction in supplies of agricultural and forestry products. Compared with other cases, this method generates very small price impacts, see Table 3.

4.4 Welfare impacts generated by alternative approaches

The welfare impacts generated by the three approaches are presented in Table 4 for four representative regions: US, EU, Brazil, and Japan.

Table 4. Welfare impacts obtained from alternative examined experiments for four representative regions (figures are in million USD)

Region	CET1	MCET or ACET	PAMC
USA	-16,122	-15,727	-16,011
EU27	1,814	2,231	2,285
BRAZIL	197	111	95
Japan	-58	269	409

Source: Author calculations.

From this table we can make the following inferences:

i) The MCET and ACET approaches generate identical welfare impacts, but their results are different from the welfare impacts of the CET1 approach.
ii) The MCET and ACET approaches may add positive or negative values to the welfare impacts provided by the CET1 approach. For example, for the case of US, the MCET and ACET similarly add +$395 million dollars to the welfare impact of CET1. For the case of Brazil, the

difference is -$56 million. For the case of Japan, the difference is about $353 million, and that changes the sign of welfare impact.

iii) The CET1 and PAMC approaches generate similar welfare impacts for USA. However, the MCET, ACET. and PAMC represent similar welfare impacts for EU27 and also for Brazil. Japan gets the largest welfare impact under the PAMC method.

To understand the source of unusual welfare impacts generated by the MCET and ACET, consider Table 5 which decomposes the welfare impacts of the CET1 and MCET simulations using the GTAP welfare decomposing program for the US economy[22]. As shown in this table, the MCET simulation generates a positive endowment effect of $1,500 million, while the CET1 simulation shows zero endowment effect.

Table 5. Welfare decomposition for US economy obtained from alternative examined experiments (figures are in million USD)

Description	Allocation effect	Endowment effect	Terms of trade effect	Investment-Saving effect	Total effect (EV)
CET1	-20,189	0	3,708	359	-16,122
MCET	-20,879	1,500	3,082	571	-15,727
PAMC	-20,670	1,405	2,648	606	-16,011

Source: Author calculations.

As mentioned earlier the CET approach assumes percentage change in productivity adjusted area of land is zero (i.e., $q = 0$). This assumption implies zero endowment effect. However, the MCET and PAMC approaches assumes $x = 0$ and allows q to vary to maintain total area of land in balance. Clearly that generates some endowment effect. To be confirmed, consider the changes in the US productivity land endowment by AEZ generated by the MCET and PAMC approaches in Figure 5. As shown in this figure for these two approaches the productivity adjusted land endowment grows across ZEZs except for small reductions in AZE15 and 15 under the PAMC. These changes show that the MCET and PAMC approach generates shifts in the land frontier, as we discussed in the analytical section. The positive endowment effects reflect converting low value lands to high value lands with no opportunity cost. The negative items show a reverse conversion. Note that the change in land endowment could have some secondary implications that alter other components of welfare. For example, the CET1 simulation represents a positive terms of trade impact by $3,708 million, while the MCET simulation projects a lower terms of trade effect of $3,082 million.

[22] The GTAP standard welfare decomposition program provides a false welfare decomposition for the ACET approach because it cannot recognize the endowment effects generated by this approach.

Clearly, the PAMC approach which generates the lowest price impacts provides the lowest terms of trade impact by $2,648.

In conclusion, compared to the traditional CET approach, the MCET and ACET which remove imbalances in land area generate the same welfare impacts induced by changes in the productivity adjusted land area. The PAMC also generates some changes in the productivity adjusted area. However, this approach generates the lowest impacts on the terms of trade due to small impacts on the prices of agricultural and forestry products.

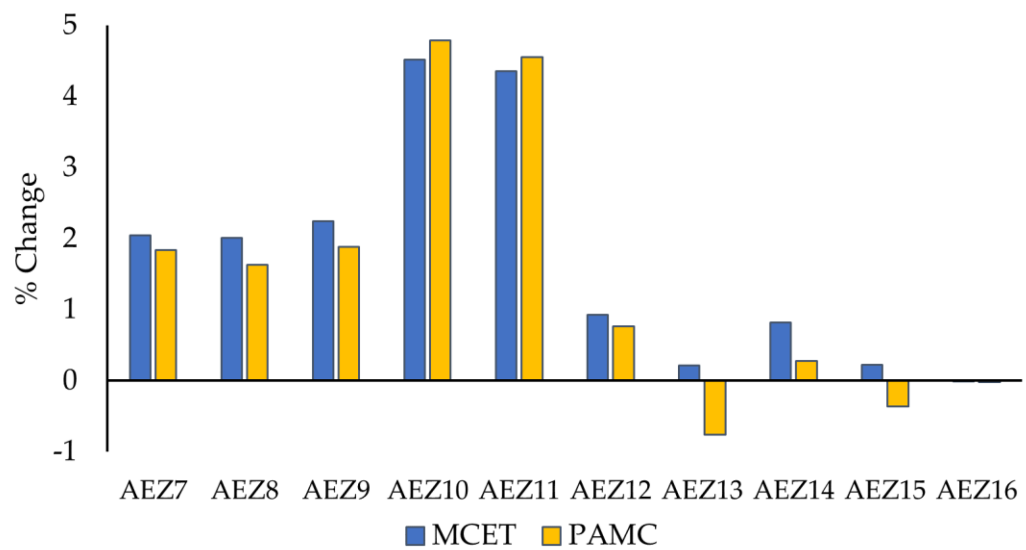

Figure 5. Change in US land endowment obtained from MCET and PAMC simulations

Source: Author calculations

5. Suggestions and conclusions

In this paper we showed that the CGE models that use the CET approach to allocate land across uses apply *ad hoc ex post* adjustment methods to keep physical area of land in balance with no side effect for welfare analyses. On the other hand, these CGE models can use *ex ante* approaches to maintain physical area of land in balance. However, these approaches undermine the opportunity costs of land transformation. In addition, all of the CET based methods (*ex post* or *ex ante*) use land transformation elasticities with limited or vague links to real observations.

We also reviewed several non-CET alternative approaches that have been used in CGE models to allocate land across uses. Some CGE models used stochastic productivity distribution functions such as Fréchet distribution These methods remove the discrepancy in land allocation and maintain area of land in balance. They also respect the conventional welfare analyses and take into account implicit costs of land conversion from on application to another one. However, in these models the production functions of the land using sectors do not represent constant returns to scale at the aggregate level. These methods also require equalization of land rents across uses in the benchmark data. Real observations are not in line with this requirement. Finally, the CGE models that use stochastic productivity distribution require a calibration process to take into account yields across uses.

Including explicit costs of land preparation and land conversion in CGE models, as followed in the case of EPPA model, is a new and promising approach. In this approach, the CGE model explicitly determines costs of land conversion and uses an optimization problem to compare the land conversion costs with its potential future rents. Gohin (2019) has followed this approach in a recent land use modeling practice. While this approach makes the land conversion more transparent, it needs data to ensure that the calculated costs of land conversion are consistent with real observations. Also, more work is needed to determine distribution of these costs over time in a dynamic model. It is not clear how these costs should be handled in a static CGE model. Finally, productivity of land in transition in this approach needs major attention.

In conclusion, our analyses indicate the existing approaches that allocate land among uses all have their own advantages and disadvantages. Clearly the CET fails to maintain area of land in balance. However, it implicitly takes into account costs of land. The non-CET approaches that use productivity distribution functions take care of physical land conversion properly, respect the conventional welfare analyses, and address implicit costs of land conversion from one use to another one. However, they have other limitations. One promising approach in modeling land use in CGE is to take into account explicit costs of land preparation and land conversion.

A short run compromise, in particular for the GTAP based models and their successors which use CET, is to use the MCET approach, acknowledging that this approach has some unconventional welfare impacts. This approach is consistent with the GTAP code and structure. One could potentially add land conversion costs to deal with the unconventional welfare impacts induced by the changes in the productivity adjusted land.

In the long run, the following line of research can help to improve modeling land use in CGE models:

i) Examine the extent to which CET functions impose costs of land transformation and how realistic they are.
ii) Collect and develop data on costs of land transformation across uses.

iii) Develop new approaches to explicitly introduce costs of land conversion based on actual data into CGE models.
iv) Develop GTAP-based models in percent change form to use stochastic productivity distribution functions.
v) To successfully and effectively incorporate land (and other biophysical data) into a CGE model we need more reliable and up to date data. In this paper we intentionally focused on the technical issues and ignored the data issues. Currently, the CGE community basically relies on the GTAP land use data base. A major effort was made to construct the first version of this data base (Lee et al., 2009; Monfreda et al., 2009; Sohngen et al., 2009) which represents the global land use 2000. While several attempts have been made to refresh this data base over time (Avetisyan et al., 2011; Baldos, 2017; Baldos and Hertel, 2012) using country level data, no major attempt has been made to update this data base at the finer resolutions, say AEZ or grid cell. The newer GTAP land use data bases basically follow the land distributions of the original data base across grid cells and AEZs. An update in this data base is required. Also, reliable data on land conversion costs and land accessibility are needed.
vi) Finally, the productivity of land in transition from one use to an alternative use is another important related topic in the land use literature. The EPPA model assumes that land in transition takes the average productivity of land in its destination, after paying costs of land conversion (Gurgel et al., 2016). This model uses a terrestrial model (TEM) to evaluate productivity measured in terms of Net Primary Products (NPP). Taheripour et al. (2012) estimated a set of extensive margins by country and AEZ, using the TEM model for the GTAP-BIO model and replaced the original simple assumption that new cropland is 2/3 as productive as the existing cropland (Hertel et al., 2010). More research is needed to evaluate productivity of land in transition.

Acknowledgement:

This research was partially funded by the National Biodiesel Foundation and the U.S. Federal Aviation Administration Office of Environment and Energy through ASCENT, the FAA Center of Excellence for Alternative Jet Fuels and the Environment, ASCENT project number 107208 through FAA Award Number 13-C-AJFE-PU under the supervision of Anna L. Oldani. Any opinions, findings, conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the FAA.

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Appendix A: Some derivations

1. Derivation of $\boldsymbol{v = \sum_j \theta_j x_j}$

Consider the following optimization problem:

Max $\sum_j P_j X_j$

Subject to: $V = \left[\sum_j \alpha_j X_j^{-\rho}\right]^{-\frac{1}{\rho}}$.

The first order conditions of this optimization problem lead to the following relationships:

$$P_j = \lambda \left[\sum_j \alpha_j X_j^{-\rho}\right]^{-\frac{(1+\rho)}{\rho}} \alpha_j X_j^{-\rho-1} \text{ for j=1,...,J} \qquad \text{A1}$$

$$\left[\sum_j \alpha_j X_j^{-\rho}\right]^{-\frac{1}{\rho}} - V = 0 \qquad \text{A2}$$

Here λ is the Lagrange coefficient of the optimization problem. Other variables are defined in the manuscript.

From A2 we can get: $V^{(1+\rho)} = \left[\sum_j \alpha_j X_j^{-\rho}\right]^{-\frac{(1+\rho)}{\rho}}$. By replacing this equation into A1 we can get:

$$P_j = \lambda V^{(1+\rho)} \alpha_j X_j^{-(\rho+1)} \qquad \text{A3}$$

Multiply both sides of A3 by X_j

$$P_j X_j = \lambda V^{(1+\rho)} \alpha_j X_j^{-\rho} \qquad \text{A4}$$

Summing over j

$$\sum_j P_j X_j = \lambda V^{(1+\rho)} \sum_j \alpha_j X_j^{-\rho} \qquad \text{A5}$$

Divide A4 by A5

$$\frac{P_j X_j}{\sum_j P_j X_j} = \frac{\alpha_j X_j^{-\rho}}{\sum_j \alpha_j X_j^{-\rho}} \qquad \text{A6}$$

The left hand side of this equation shows the revenue share of land in use j, θ_j. Therefore:

$$\theta_j = \frac{P_j X_j}{\sum_j P_j X_j} = \frac{\alpha_j X_j^{-\rho}}{\sum_j \alpha_j X_j^{-\rho}} \qquad \text{A7}$$

Total derivative of the CET land frontier of $V = \left[\sum_j \alpha_j X_j^{-\rho}\right]^{-\frac{1}{\rho}}$ provides:

$$dV = \left(-\frac{1}{\rho}\right)\left[\sum_j \alpha_j X_j^{-\rho}\right]^{-\frac{(1+\rho)}{\rho}} (-\rho)\alpha_1 X_1^{-\rho-1} dX_1 + \cdots + \left(-\frac{1}{\rho}\right)\left[\sum_j \alpha_j X_j^{-\rho}\right]^{-\frac{(1+\rho)}{\rho}} (-\rho)\alpha_J X_J^{-\rho-1} dX_J \qquad \text{A8}$$

The above equation can be simplified as:

$$dV = \left[\sum_j \alpha_j X_j^{-\rho}\right]^{-\frac{(1+\rho)}{\rho}} \left[\alpha_1 X_1^{-\rho-1} dX_1 + \cdots + \alpha_J X_J^{-\rho-1} dX_J\right] \qquad \text{A9}$$

Divide both sides by V:

$$\frac{dV}{V} = \frac{\left[\sum_j \alpha_j X_j^{-\rho}\right]^{-\frac{(1+\rho)}{\rho}} \left[\alpha_1 X_1^{-\rho-1} dX_1 + \cdots + \alpha_J X_J^{-\rho-1} dX_J\right]}{\left[\sum_j \alpha_j X_j^{-\rho}\right]^{-\frac{1}{\rho}}} \qquad \text{A10}$$

One can use simple algebra and rewrite the above equation in the following form:

$$\frac{dV}{V} = \frac{\left[\alpha_1 X_1^{-\rho} \frac{dX_1}{X_1} + \cdots + \alpha_J X_J^{-\rho} dX_J\right]}{\left[\sum_j \alpha_j X_j^{-\rho}\right]} \qquad \text{A11}$$

Following our notation from the manuscript: $\frac{dV}{V} = v$ and $\frac{dX_j}{X_j} = x_j$. Therefore:

$$v = \frac{\left[\alpha_1 X_1^{-\rho}\right]}{\left[\sum_j \alpha_j X_j^{-\rho}\right]} x_1 + \cdots + \frac{\left[\alpha_1 X_1^{-\rho}\right]}{\left[\sum_j \alpha_j X_j^{-\rho}\right]} x_j \qquad \text{A12}$$

Combine equations A7 and A12 and find the final results:

$$v = \theta_1 x_1 + \cdots + \theta_j x_j = \sum_j \theta_j x_j \qquad \text{A13}$$

2. *Derivation of land supply for CET approach:*

For simplification and without loss of generality assume that there are only two types of land: j=1 and j=2. Write equation A3 for j=1 and j=2 and dived the first equation by the second one. That provides the following relationship:

$$\frac{P_1}{P_2} = \frac{\lambda V^{(1+\rho)} \alpha_1 X_1^{-(\rho+1)}}{\lambda V^{(1+\rho)} \alpha_2 X_2^{-(\rho+1)}} \qquad \text{A14}$$

With some simplification we can get:

$$\frac{P_1}{P_2} = \frac{\alpha_1 X_1^{-(\rho+1)}}{\alpha_2 X_2^{-(\rho+1)}} \qquad \text{A15}$$

Solve for $X_2$and do some regular algebra work:

$$X_2 = \left(\frac{P_1}{\alpha_1}\right)^{\frac{1}{\rho+1}} \left(\frac{P_2}{\alpha_2}\right)^{\frac{-1}{\rho+1}} X_1 \qquad \text{A16}$$

Raise the power of A16 by $-\rho$

$$X_2^{-\rho} = \left(\frac{P_1}{\alpha_1}\right)^{\frac{-\rho}{\rho+1}} \left(\frac{P_2}{\alpha_2}\right)^{\frac{\rho}{\rho+1}} X_1^{-\rho} \qquad \text{A17}$$

Put equation A17 into equation A2 (the land frontier):

$$V = \left(\alpha_1 X_1^{-\rho} + \alpha_2 \left(\frac{P_1}{\alpha_1}\right)^{\frac{-\rho}{\rho+1}} \left(\frac{P_2}{\alpha_2}\right)^{\frac{\rho}{\rho+1}} X_1^{-\rho}\right)^{-1/\rho} \qquad \text{A18}$$

Solve for X_1:

$$X_1 = \frac{V}{\left(\alpha_1 + \alpha_2 \left(\frac{P_1}{\alpha_1}\right)^{\frac{-\rho}{\rho+1}} \left(\frac{P_2}{\alpha_2}\right)^{\frac{\rho}{\rho+1}}\right)^{-\frac{1}{\rho}}} \qquad \text{A19}$$

With some rearrangement work we can rewrite the above function in the following form:

$$X_1 = \frac{\left(\frac{P_1}{\alpha_1}\right)^{\frac{-1}{\rho+1}}}{\left(\alpha_1\left(\frac{P_1}{\alpha_1}\right)^{\frac{\rho}{\rho+1}}+\alpha_2\left(\frac{P_2}{\alpha_2}\right)^{\frac{\rho}{\rho+1}}\right)^{-\frac{1}{\rho}}}(V) \qquad \text{A20}$$

Following the symmetry property we can write the following general supply function each use, when there are multiple uses of land.

$$X_j = \frac{\left(\frac{P_j}{\alpha_j}\right)^{\frac{-1}{\rho+1}}}{\left(\sum_j \alpha_j\left(\frac{P_j}{\alpha_j}\right)^{\frac{\rho}{\rho+1}}\right)^{-\frac{1}{\rho}}}(V) \qquad \text{A21}$$

It is straight forward to show that the CET land supply function in the percent change is:

$$x_j = v - \sigma p_j + \sigma \sum_j \theta_j p_j \qquad \text{A22}$$

3. *Derivation of land supply for ACET approach:*

To derive the ACET land supply function we follow the optimization problem defined by van der Mensbrugghe and Peters (2016). Adjusting their model with our assumptions and variable names provides the following optimization problem:

Max $U = \left[\sum_j \alpha_j X_j^{-\rho}\right]^{-\frac{1}{\rho}}$
Subject to: $\mathrm{X} = \sum_j X_j$.

Similar to the case of CET here, $\rho \leq -1$. The first order conditions of this optimization problem implies:

$$\alpha_j P_J^{-\frac{1}{\rho}} X_j^{-\frac{1}{\rho}-1}\left[\sum_j \alpha_j \left(P_j X_j\right)^{-\frac{1}{\rho}}\right]^{-\rho-1} = \lambda \qquad \text{A23}$$

Which implies

$$X_j = \alpha_j^{\frac{\rho}{1+\rho}} P_j^{-\frac{1}{1+\rho}}\left[\sum_j \alpha_j \left(P_j X_j\right)^{-\frac{1}{\rho}}\right]^{-\rho} \lambda^{-\frac{\rho}{1+\rho}} \qquad \text{A24}$$

Using the land constraint condition, $\mathrm{X} = \sum_j X_j$, implies:

$$\left[\sum_j \alpha_j \left(P_j X_j\right)^{-\frac{1}{\rho}}\right]^{-\rho} \lambda^{-\frac{\rho}{1+\rho}} = \frac{X}{\sum_i \alpha_j^{\frac{\rho}{1+\rho}} P_j^{-\frac{1}{1+\rho}}} \qquad \text{A25}$$

Using equation A24 implies:

$$\lambda^{-\frac{\rho}{1+\rho}} = \frac{X_j}{\alpha_j^{\frac{\rho}{1+\rho}} P_j^{-\frac{1}{1+\rho}} \left[\sum_j \alpha_j (P_j X_j)^{-\frac{1}{\rho}}\right]^{-\rho}} \qquad \text{A26}$$

Using equations A25 and A26 provides:

$$X_j = \frac{(\alpha_j)^{\frac{1}{1+\rho}} (P_j)^{\frac{-\rho}{1+\rho}}}{\sum_j (\alpha_j)^{\frac{1}{1+\rho}} (P_j)^{\frac{-\rho}{1+\rho}}} (X) \qquad \text{A27}$$

The ACET land supply function in the percent change is:

$$x_j = x - \sigma p_j + \sigma \sum_j \phi_j p_j \qquad \text{A28}$$

4. *Comparison of land supply functions of CET and ACET approaches:*

One cannot easily provide a mapping relationship between the land supply functions of CET and ACET with level variables (i.e. equations A21 and A27) for three reasons: a) the relationship between V and X is not known; b) for a given land transformation elasticity, the exponent values of CET and ACET are not identical; 3) the values of distribution parameters of CET and ACET are not identical. For more discussion see van der Mensbrugghe and Peters (2016).

While it is not feasible to establish a mapping relationship between the land supply functions of CET and ACET with level variables, Zhao et al. (2020b) have shown that in percent change an ACET land supply function can be decomposed into a CET land supply and a shifting factor.

Furthermore, the price link in both approaches can be derived using the zero-profit condition. It is important to note that in the conventional CET derivation, the price link obtained from the zero-profit condition was simplified to $p = \sum_i \theta_i p_i$ owing to the condition of $x = \sum_i \theta_i x_i$ implied by profit maximization in the CET approach. However, because $x = \sum_i \theta_i x_i$ does not hold in the ACET approach, full terms in the zero-profit condition are needed to represent a quantity-share-weighted-price link. The price link change made for ACET was reflected in the supplementary model code in both land supply and welfare decomposition.

Appendix B: Land allocation using extreme value distribution functions

In this appendix, we present a stylized model of land use based on discrete choices of agricultural producers whose land productivities are heterogeneous with a distribution function that is Extreme Value Type II or Fréchet. What we present here is only the production side in its simplest way, which could be extended in multiple ways, and combined with a demand side to constitute an equilibrium.

1. *A Stylized model of Fréchet-based land use*

Consider a stylized production side of an economy: production technology is constant returns to scale at the level of individual producers, land is the only factor of production, and markets are perfectly competitive. The set of goods consists of crops[23] indexed by $j \in J$. Land consists of a continuum of plots $\omega \in [0, 1]$. At every plot, an agriculture producer faces a discrete choice problem of selecting the crop that maximizes returns to that given plot. Aggregating plot-level decisions give rise to aggregate land allocated to crop j, denoted by X_j, and aggregate supply of crop j, denoted by Q_j. Total physical area of land is denoted by X, and the market price of crop j is denoted by P_j.

In every plot ω, sales from crop $j \in J$ is given by $P_j Z_j(\omega)$ where $Z_j(\omega)$ is a plot-specific land productivity. A higher P_j relative to $P_{j'}$ for all $j' \neq j$ signals the market demand for j. A higher $z_j(\omega)$ relative to $z_{j'}(\omega)$ for all $j' \neq j$ reflects the higher suitability of plot ω for j. Hence, $P_j Z_j(\omega)$ summarizes the combined effect of market signals and natural suitability.

Without loss of generality, the area of every plot is normalized to one. Then, $Z_j(\omega)$ represents yield of crop j, and $P_j Z_j(\omega)$ is returns to land in plot ω if crop j is selected. The problem of land use allocation in plot ω is then to maximize returns to land across alternatives:

$$\max \{P_j Z_j(\omega) \ \ \forall\, j \in J\}$$

This problem has a closed-form solution if $Z_j(\omega)$ follows an Extreme Value Type II (Fréchet) distribution.[24] Formally, let $Z_j(\omega)$ be a random variable drawn independently across ω according to:

$$F_j(Z_j) \equiv \Pr\big(Z_j(\omega) \le Z_j\big) = \exp\left(-\phi\left({}^{Z_j}\!/_{a_j}\right)^{-\varphi}\right) \quad \text{for all } Z_j \ge 0 \qquad \text{B1}$$

Here, $\phi = \Gamma(1 - \frac{1}{\varphi})$ is a constant normalizer to ensure that $\mathbb{E}\big[Z_j(\omega)\big] = a_j$ as the unconditional average land productivity of crop j over the entire space of plots.[25]

[23] The set of land using goods can be extended to crops, pasture, and forest.

[24] The connection to the Gumbel distribution can be derived by a transformation of PZ to $\ln(PZ)$ and allowing $\ln(Z)$ to have a Gumbel (Type I Extreme Value) distribution.

[25] $\Gamma(.)$ is the gamma function, and $\mathbb{E}\big[Z_j(\omega)\big] = \int_0^\infty Z\, dF_j(Z)$.

In addition, $\varphi \geq 1$ controls the dispersion across productivity draws. The smaller φ is, the larger the variance of productivity draws, the more heterogeneity in crop suitability across plots in the space of land.[26]

2. *Land use*

Let X be total land which is exogenously given, and X_j be land use of crop j. Let Ω_j denote the set of plots in which j maximizes returns, and $\Pi_j = \Pr(\omega \in \Omega_j)$ be the probability that crop j be rent-maximizing. Using the properties of the Fréchet distribution:

$$\Pi_j \equiv \frac{X_j}{X} = \frac{(P_j a_j)^{\varphi}}{\sum_{j \in J}(P_j a_j)^{\varphi}} \qquad \text{B2}$$

Because there is a continuum of plots, by the Law of Large Number, Π_j is both the probability of selecting j for a plot, and the share of land allocated to crop j as the aggregation of discrete choices of agricultural producers. Here, φ can be thought of as the elasticity of land use X_j with respect to market price P_j (or average suitability a_j).

3. *Crop output*

The aggregate yield of crop j equals the averages of $Z_j(\omega)$ only for plots to which crop j is allocated, $\omega \in \Omega_j$. Again, using properties of the Fréchet distribution:

$$\mathbb{E}\left[Z_j(\omega) | \omega \in \Omega_j\right] \equiv \int_{Z \in \Omega_j} Z \, dF_j(Z) = a_j \Pi_j^{-\frac{1}{\varphi}} \qquad \text{B3}$$

The aggregate yield, or the conditional mean of land productivity, depends on suitability parameter a_j adjusted for the margin of selections. The conditional mean of yield of j is necessarily greater than the unconditional mean:

$$\mathbb{E}\left[Z_j(\omega) | \omega \in \Omega_j\right] = a_j \Pi_j^{-1/\varphi} \geq \mathbb{E}\left[Z_j(\omega)\right] = a_j \qquad \text{B4}$$

Underlying to this inequality is the land heterogeneity with the extent of the relationship governed by parameter φ. If $\varphi \to \infty$, then land is not heterogeneous anymore, and whatever maximizes rents in one plot will also maximizes rents elsewhere. This is an extreme case in which the entire land would be allocated to one crop, and conditional and unconditional average land productivities would be the same.

However, in the general case, agriculture producers select the plots for a crop that are more suitable with respect to that crop. For example, when the price of a crop falls, agriculture producers will be more selective in the use of land for that crop in order to maintain profitability. We refer to this channel as "selection". The

[26] Precisely, the standard deviation of $\ln Z_j(\omega)$ is proportional to $1/\varphi$.

term, $\Pi_j^{-\frac{1}{\varphi}}$, as the endogenous part of land productivity reflects this selection margin. Here, land share is in fact a sufficient statistic for learning about the endogenous changes to land productivity. The extent to which this channel increases yields is governed by φ. Again, a lower φ reflects a greater heterogeneity of suitability across plots, which in turn implies a larger gain in yields due to selections. Aggregate output of crop j then equals:

$$Q_j = X\Pi_j \, \mathbb{E}\left[Z_j(\omega)|\omega \in \Omega_j\right] = Xa_j\Pi_j^{(\varphi-1)/\varphi} \quad \text{B5}$$

where X is total land area. An increase in land share of j, Π_j, increases production quantity, Q_j, less than proportional due to the opportunity cost of giving up producing previously suitable alternatives. In this sense, $(\varphi - 1)/\varphi$ appears as the elasticity of output with respect to land share.

Notice that production function is constant returns to scale (homogenous of degree 1) at the level of individual producers, but it is homogenous of degree $\frac{\varphi-1}{\varphi} < 1$ at the aggregate level. We decompose these channels of effects:

$$\begin{aligned}\underbrace{\Delta \ln Q_j}_{\%\ cha.\ in\ supply} &= \underbrace{\Delta \ln\left(X_j\right)}_{\%\ cha.\ in\ land\ use} + \underbrace{\Delta \ln(Land\ Productivity)}_{\%\ cha.\ in\ productivity} \\ &= \underbrace{\Delta \ln(\Pi_j)}_{\%\ cha.\ in\ land\ share} + \underbrace{\Delta \ln(a_j)}_{\%\ cha.\ in\ exog.\ productivity} + \underbrace{(-1/\varphi)\Delta \ln(\Pi_j)}_{\%\ cha.\ in\ endog.\ productivity}\end{aligned} \quad \text{B6}$$

where the first term in the RHS changes output due to a change in land use, and the second term in the RHS is the effect due to productivity change. The productivity change has two components. First, it is trivially due to an exogenous change (such as climate). Second, and most importantly, productivity changes endogenously. The Fréchet structure implies that the change to land share $\Delta \ln \Pi_j$ with elasticity $\varphi > 1$ are sufficient statistics for the endogenous adjustment to land productivity.

4. Rents

Let $R_j(\omega)$ denote rents collected in plot $\omega \in \Omega_j$. Since markets are perfectly competitive, profits are pushed down to zero, and so, $R_j(\omega) = P_jZ_j(\omega)$. Aggregate rents obtained from crop j, denoted by $\hat{R}_j$, equals:

$$\hat{R}_j = X\Pi_j \, \mathbb{E}\left[P_jZ_j(\omega)|\omega \in \Omega_j\right] = X\,P_ja_j\Pi_j^{(\varphi-1)/\varphi} \quad \text{B7}$$

Rents from j are higher due to land use of crop j, $X_j = X\Pi_j$, price of output P_j, average suitability of land for crop j reflected by a_j, all adjusted for the selection margin $\Pi_j^{-1/\varphi}$. We could equivalently obtain rents using the accounting: $\hat{R}_j = P_j Q_j$.

One implication of the rent equation is that rent per unit of land equalizes across uses. Specifically, for every $j \in J$:

$$\frac{\hat{R}_j}{X_j} = \bar{R} = \left[\sum_{j\in J}\left(p_j a_j\right)^{\varphi}\right]^{1/\varphi} \qquad \text{B8}$$

5. Extensions

Here, we review a summary of extensions to allow for multiple factors of production, extensive margin of cropland, and nested structure allowing for flexible choice margins.

Consider a general production function:

$$Q_j(\omega) = \min\left(f\left(Z_j(\omega)X_j(\omega), N_j(\omega), M_j(\omega)\right), F_0(\omega)/Z_0(\omega)\right) \qquad \text{B9}$$

Here, $Q_j(\omega)$ is crop production in plot ω, and $X_j(\omega)$, $N_j(\omega)$, $M_j(\omega)$ are respectively use of land, labor, and intermediate material. Now, every plot is endowed not only by land productivity draws $(Z_1(\omega), \dots, Z_J(\omega))$ but also by an investment intensity draw $Z_0(\omega)$. $Q_j(\omega)$ is by structure a Leontief combination of variable production and upfront investment, meaning that the agricultural producer pays a fixed investment cost in order to set up plot ω for any crop cultivation. Assuming that the unit cost of investment is P_0, returns to plot ω are given by:

$$P_j\, f\left(Z_j(\omega)X_j(\omega), N_j(\omega), M_j(\omega)\right) - P_0 Z_0(\omega) \qquad \text{B10}$$

This extension allows for an endogenous margin of cropland. Specifically, for every plot ω in which the fixed costs exceed returns to every crop, the plot remains unused. In this case, the sum of land share of crops and non-cropland will be equal to one. Then, for instance, if crop prices rise relative to the cost of investment, cropland expands and non-cropland shrinks.

Here, $f\left(Z_j(\omega)X_j(\omega), N_j(\omega), M_j(\omega)\right)$ is any function that is homogenous of degree one, (CES, Cobb-Douglas, etc). In Costinot, Donaldson, and Smith (2016) and Gouel and Laborde (2018), $f = z_j(\omega)X_j(\omega)$ and the fixed costs are paid to labor. In Sotelo (2019), $f = \left(Z_j(\omega)X_j(\omega)\right)^{\gamma_L}\left(N_j(\omega)\right)^{\gamma_N}\left(M_j(\omega)\right)^{\gamma_M}$, $\gamma_L + \gamma_N + \gamma_M = 1$, and fixed costs are zero.

Appendix C: Numerical analyses using CET, MCET and Fréchet stylized models

In this appendix, we present three stylized CGE models of land use based on three specification of CET, Modified CET (MCET), and Fréchet. The appendix is standalone in the sense that it can be studied with or without the main body of the paper. However, in the paper we have discussed the main takeaways from this appendix.

In Section 1 of this appendix, we present three stylized models that feature the same consumption side but follow different production sides. Corresponding to each production side, we then define a competitive equilibrium. In Section 2, we show how to calibrate each of these resulting equilibrium models to the same base data on land uses and crop quantities. In Section 3, we present numerical examples that demonstrate the predictions of each of these models in response to a common counterfactual policy. We highlight where these models are equivalent and where they go different, and provide a theoretical insight behind the results.

1. Setup

Consider a stylized economy in which land is the only factor of production and markets are perfectly competitive. The set of goods consists of crops indexed by $j \in J$. In what follows, we first present a simple consumption side, then three production sides based on CET, MCET, and Fréchet. Then, we demonstrate the precise definition of equilibrium for each.

1.1. Consumption

Consumers' utility derived from consumption of crops $j \in J$, $\{C_j\}$, is given by a CES aggregator:

$$U = \left[\sum_{j\in J}\left(b_j\right)^{1/\partial} C_j^{\frac{\partial-1}{\partial}}\right]^{\partial/(\partial-1)} \qquad \text{C1}$$

where b_j is a consumption shifter and $\sigma > 0$ is the elasticity of substitution across crops. Consumption quantity of crop $j \in J$ is then given by:

$$C_j = \frac{b_j\left(\tau_j P_j\right)^{-\partial}}{\sum_{j\in J} b_j\left(\tau_j P_j\right)^{1-\partial}} Y \qquad \text{C2}$$

Where P_j is price of crop j, τ_j is tax ($\tau_j > 1$) or subsidy ($\tau_j < 1$) on the consumption of crop j, and Y is total income.

1.2. Production

We denote by R_j the price (rent) of land for use j and by P_j the (before-tax) price of crop j. Let X_j be the physical land use allocated to crop j and Q_j be supply quantity of crop j. Total physical land endowment is denoted by $X = \sum_{j\in J} X_j$. In what follows, for each of the three production sides, we show the supply equation of land use X_j, yield Y_j, and crop output Q_j. Note that X_j and Q_j immediately imply yield as $Y_j = Q_j/X_j$. The emphasis on yield is merely to convey the intuition behind differences across these models.

1.2.1. Constant elasticity transformation (CET)

The land allocation problem involves maximizing total rents, $\mathrm{R} = \sum_{j\in J} R_j X_j$, subject to the CET constraint:

$$V = \left[\sum_j \alpha_j^{CET} X_j^{\rho}\right]^{1/\rho} \quad \text{C3}$$

where $\rho > 1$. α_j is a shifter, and V represents the exogenous endowments of efficiency units of land. Physical land allocated to crop j as a function of land rent R_j is given by:

$$X_j = \frac{\left(\frac{R_j}{\alpha_j^{CET}}\right)^{\frac{1}{\rho-1}}}{\left(\sum_j \alpha_j^{CET}\left(\frac{R_j}{\alpha_j^{CET}}\right)^{\frac{\rho}{\rho-1}}\right)^{1/\rho}} V \quad \text{C4}$$

Yield of crop j, denoted by Y_j, is exogenously given by a_j^{CET},

$$Y_j = a_j^{CET} \quad \text{C5}$$

Therefore, provided that land use of crop j is positive, $Q_j = a_j^{CET} X_j$. In addition, since markets are perfectly competitive and profits are pushed down to zero, $R_j = a_j^{CET} P_j$. Putting these together:

$$Q_j = \frac{\left(\gamma_j^{CET}\right)^{\frac{\rho}{\rho-1}}\left(P_j\right)^{\frac{1}{\rho-1}}}{\left(\sum_j\left(\gamma_j^{CET}\right)^{\frac{\rho}{\rho-1}}\left(P_j\right)^{\frac{\rho}{\rho-1}}\right)^{1/\rho}} \times V \quad \text{C6}$$

where $\gamma_j^{CET} = a_j^{CET}\left(\alpha_j^{CET}\right)^{-1/\rho}$.

1.2.2. MCET (Modified CET)

Consider the same triple of equations C4-6 for land use, yield, and output of crops, along with $R_j = a_j^{CET} P_j$. In addition, here we add a constraint on the sum of land use across crops:

$$\sum_{j\in J} X_j = X \quad \text{C7}$$

The difference with the CET model is that total efficiency units of land, V, is now endogenous whereas total physical land, X, is taken as exogenous.

1.2.3. Fréchet

As we present a detailed derivation in Appendix B, land use X_j, yield Y_j, and output Q_j of every crop j are given by:

$$X_j = \frac{\left(P_j a_j^{FRE}\right)^{\varphi}}{\sum_{j\in J}\left(P_j a_j^{FRE}\right)^{\varphi}} X \qquad \text{C8}$$

$$Y_j = a_j^{FRE}\left(\frac{\left(P_j a_j^{FRE}\right)^{\varphi}}{\sum_{j\in J}\left(P_j a_j^{FRE}\right)^{\varphi}}\right)^{-1/\varphi} \qquad \text{C9}$$

$$Q_j = a_j^{FRE}\frac{\left(P_j a_j^{FRE}\right)^{\varphi-1}}{\left(\sum_{j\in J}\left(P_j a_j^{FRE}\right)^{\theta}\right)^{\frac{\varphi-1}{\varphi}}} X \qquad \text{C10}$$

where $\varphi > 1$, and a_j^{FRE} is a shifter. Total physical land X is the exogenous endowment in the economy. Equation C9 guarantees that $\sum_{j\in J} X_j = X$.

1.3. Equilibrium

Market clearing condition requires consumption and production of every crop to be equal:

$$Q_j = C_j \qquad \text{C11}$$

and, total income be given by:

$$Y = \sum_j \tau_j P_j Q_j \qquad \text{C12}$$

Here are the equilibrium definitions for the three models:

CET: Given demand parameters $\{b_j, \partial\}$, production parameters $\{\alpha_j^{CET}, a_j^{CET}, \rho\}$, endowment V, and policy $\{\tau_j\}$, an equilibrium consists of prices $\{P_j\}$ such that C_j is given by C2, Q_j is given by C6, and market clearing conditions C11 and C12 hold. Land use X_j is given by C4, and yield Y_j is exogenously given by C5.

MCET: Given demand parameters $\{b_j, \partial\}$, production parameters$\{\alpha_j^{CET}$, $a_j^{CET}, \rho\}$, endowment X, and policy $\{\tau_j\}$, an equilibrium consists of prices $\{p_j\}$ such that C_j is given by C2, Q_j is given by C6, and market clearing conditions C11 and C12 hold. Land uses $\{X_j\}$ and V satisfy C4 and constraint C7. Yield Y_j is exogenously given by C5.

Fréchet - Given demand parameters $\{b_j, \sigma\}$, production parameters $\{a_j^{FRE}, \theta\}$, endowment X, and policy $\{\tau_j\}$, an equilibrium consists of prices $\{p_j\}$ such that C_j is given by C2, Q_j is given by C10, and market clearing conditions C11 and C12 hold. Land use X_j is given by C8, and yield Y_j is endogenously given by C9.

2. Calibration

Suppose we observe the vector of crop quantities $\{Q_j\}$ and the vector of land uses $\{X_j\}$ for all $j \in J$. We demonstrate how to calibrate each of these models to ensure that they reproduce in their baseline equilibrium the base data $\{Q_j\}$ and $\{X_j\}$. Notice that in the base data, by construction total physical land is given by $X = \sum_{j\in J} X_j$.

In addition, the given elasticities are: σ as the elasticity of substitution in demand, ρ as the land governance parameter in CET (or MCET); and θ for Fréchet. It is straightforward to show that ρ in CET (or MCET) and θ in Fréchet has to have the following relationship: $\rho = \frac{\varphi}{\varphi-1}$.

To generate the same curvature of production possibility frontiers across crops. We borrow $\partial = 3$ and $\varphi = 2.5$ in line with the estimates in Costinot, Dolandson, and Smith[27], and according to the above relationship, we set $\rho = \frac{2.5}{1.5}$. Recall that ∂ is the elasticity of substation across crops on the demand side, and φ or ρ effectively govern substitution patterns in production. We normalize $b_1 = 1$, and by choosing crop 1 as the numeraire, $P_1 = 1$.

2.1. Calibration of Fréchet

Production shifters are calibrated as:

$$a_j^{FRE} = \frac{Q_j}{X\left(\frac{X_j}{X}\right)^{\frac{\varphi-1}{\varphi}}} \quad \text{C13}$$

Given the above and that $P_1 = 1$, implied equilibrium prices are then known:

$$P_j = \left(\frac{X_j}{X_1}\right)^{1/\varphi} \left(\frac{a_j^{FRE}}{a_1^{FRE}}\right)^{-1} \quad \text{C14}$$

Given the implied prices and that $b_1$equals one, consumption shifters are then calibrated as:

$$b_j = \frac{Q_j}{Q_1} P_j^{\partial} \quad \text{C15}$$

2.2. Calibration of CET and MCET

We keep consumption shifters b_j exactly as the ones which we calibrated based on Fréchet. This gives us exactly the same demand system.

Yields are exogenous, and simply given by:

$$a_j^{CET} = Q_j/X_j \quad \text{C16}$$

It remains to calibrate α_j and V. Since b_j is known here, implied equilibrium prices are:

$$P_j = \left(\frac{b_j}{b_1}\right)^{\frac{1}{\sigma}} \left(\frac{Q_j}{Q_1}\right)^{\frac{-1}{\sigma}} \quad \text{C17}$$

We normalize $\alpha_1 = 1$. Hence, the composite shifter, $a_j^{CET}\left(\alpha_j^{CET}\right)^{-1/\rho}$, is known for $j = 1$. Using the CET crop supply function, γ_j^{CET} is calibrated for all j:

$$\gamma_j^{CET} = \left(\frac{Q_j}{Q_1}\right)^{\frac{\rho-1}{\rho}} \left(\frac{P_j}{P_1}\right)^{\frac{-1}{\rho}} \quad \text{C18}$$

This means that the following ratio in equation C6 will be known:

[27] These authors have estimated that: $\partial = 2.82$ and $\varphi = 2.46$

$$\lambda_j \equiv \frac{\left(\gamma_j^{CET}\right)^{\frac{\rho}{\rho-1}}(p_j)^{\frac{1}{\rho-1}}}{\left(\Sigma_j\left(\gamma_j^{CET}\right)^{\frac{\rho}{\rho-1}}(p_j)^{\frac{\rho}{\rho-1}}\right)^{1/\rho}} \tag{C19}$$

Now, calibrate V:

$$V = Q_j/\lambda_j \tag{C20}$$

By structure of our calibration, the above ratio is the same across all j. Lastly note that the calibration of parameters in CET and MCET is exactly the same. That is, in the baseline CET we have the balance of physical land. What differs between CET and MCET is changes to variables from the baseline in response to a policy shock.

3. Numerical Results

We consider a simple numerical exercise in which there are three crops: corn, soybean, and others. We calibrate the three models to the base data on land use and output quantities of these crops in the United States in 2016. The calibration parameters as well as the implied equilibrium prices, are reported in Table C1.

To numerically examine the responses of the three defined production sides with respect to an exogenous shock, we specifically consider a 20% subsidy on corn consumption, and solve the new equilibrium in each of the three models. Reported in Table C2 are exact percentage changes to prices, quantities, and welfare across the three models.

The first observation is the equivalence of Fréchet and CET in their prediction for crop price and quantities, P_j and Q_j. Since utility combines quantities, welfare implications of Fréchet and CET will be also necessarily the same.

In addition, as expected, yields endogenously change in the case of Fréchet whereas it remains exogenously unchanged in CET and MCET. The change in yields in Fréchet, and the reduction in total physical land in CET imply an exact change along the production possibility frontier of crop outputs, giving rise to their same predictions of crop output and prices. In contrast, the production possibility frontier shifts in the case of MCET making its predictions different from the other two. The MCET results are closer to the CET results. One other expected observation is that in Frehcet-based model, land rents equalize across uses.

More importantly, MCET implies a lower reduction in welfare. The underlying reason is the expansion of efficiency units V. The percentage increase of V in MCET equals the percentage decrease of X in CET. Specifically, one can check that:

$$\Delta \ln U^{MCET} - \Delta \ln U^{CET} = \Delta \ln V^{MCET} = -\Delta \ln X^{CET} \tag{C21}$$

Table C1. Calibrated Parameters

Observed Data	
Based on USA aggregate agriculture data, 2016	
Crop land use (million ha)	
$X = [37, 34, 33]$	
Crop output quantity (million ton)	
$Q = [390, 117, 253]$	
Calibrated Demand Parameters	
$b = [1.00, 8.62, 1.68]$	
$\sigma = 3$	
Calibrated Supply Parameters	
Fréchet	CET, MCET
$a^{FRE} = [6.97, 2.20, 4.84]$	$a^{CET} = [10.54, 3.44, 7.66]$,
$\varphi = 2.5$	$\gamma^{CET} = [1.00, 0.31, 0.69]$
	$\rho = \frac{\varphi}{\varphi - 1} = 1.6666 \ldots$
	$V = 735.04$
Implied Prices	
Crop	
$P = [1.00, 3.06, 1.37]$	
Land (rent)	
$R = [10.54, 10.54, 10.54]$	

Table C2. The effects from 20 percent subsidy on the consumption of corn on quantities, prices, and welfare –reported in percentage change from the baseline

Description	Fréchet	CET	MCET
Crop output quantity (%)	$[14.33, -8.53, -8.53]$	$[14.33, -8.53, -8.53]$	$[14.79, -8.16, -8.16]$
Crop land use (%)	$[25.01, -13.81, -13.81]$	$[14.33, -8.53, -8.53]$	$[14.79, -8.16, -8.16]$
Yield (%)	$[-8.54, 6.12, 6.12]$	$[0, 0, 0]$	$[0, 0, 0]$
Crops output price	$[0, -13.82, -13.82]$	$[0, -13.82, -13.82]$	$[0, -13.82, -13.82]$
Crops land rent	$[-8.54, -8.54, -8.54]$	$[0, -13.82, -13.82]$	$[0, -13.82, -13.82]$
Total land efficiency, V (%)	NA	0	0.40
Total land use (%)	0	-0.40	0
Welfare (%)	-0.59	-0.59	-0.19