

The Standard GTAP Model in GAMS, Version 7

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The purpose of this paper is to describe a version of the Global Trade Analysis Project (GTAP) computable general equilibrium (CGE) model implemented in the General Algebraic Modeling System (GAMS) that is a literal implementation of the standard General Economic Modeling Package (GEMPACK) version. There are a number of GAMS-based models that rely on the basic GTAP Data Base structure, but none is an exact replica of the GTAP model itself. This paper relies on version 7.0 of the GTAP model released in 2017 and that represents the latest version of the 'standard' GTAP model, acknowledging that there are multiple variants. The code has been tested with release 9 of the GTAP Data Base that has 57 commodities and 140 countries/regions.

JEL codes: C68, D58, D60, F1

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1. Introduction

Tom Hertel and colleagues at the Center for Global Trade Analysis (GTAP) at Purdue University started development of the so-called Global Trade Analysis Project (GTAP) model in the early 1990s in parallel with the development of the GTAP Data Base—the first widely available and public database of global economic activity designed for global computable general equilibrium (CGE) models.¹ The model, database and applications were described in detail in Hertel (1997). The GTAP model has become a *de facto* standard for global computable general equilibrium (CGE) modeling, aided to some extent by the tools developed to ease the use of the model and interpretation of the results, and the extensive instructional support provided by the Center. The standard GTAP model was developed in the Johansen style (Johansen, 1960), i.e. in percentage change form and is implemented using the General Economic Modeling Package (GEMPACK) suite of software de-

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¹The acronym GTAP is used to identify both the GTAP Center at Purdue as well as the model and database.

signed for solving CGE models (Harrison and Pearson, 1996).² One aim of this article is to provide a different entry point into the widely known and used standard GTAP model, particularly for those economists coming from the General Economic Modeling Package (GAMS) tradition of CGE modeling.³ The GAMS version of the standard GTAP model described herein is available in the supplementary files published with this article.

The standard GTAP model has been the object of a major revision (Corong et al., 2017), with version 7.0 supplanting the previous version 6.2a.⁴ This paper provides a full specification of the latest version of the standard GTAP model. It is intended to complement the GEMPACK version of the model as developed in Corong et al. (2017) including all of the extensions to the standard model since 1997. This is not the first publicly available GAMS-based model using the GTAP Data Base. For a number of years, Tom Rutherford has generously made available his own version of a GAMS-based GTAP model (Lanz and Rutherford, 2016).⁵ The model described herein differs from the Rutherford version in a number of ways. First, it is intended to be a full blown translation of GEMPACK's TABLO code of the standard model. The Rutherford version is a variant that captures most of the structural features of the standard GTAP model, but not all. Second, the model is written and coded in the Dervis, de Melo and Robinson (Dervis et al., 1982) tradition and departs significantly from the nomenclature used in the GEMPACK version of the model and that used by Rutherford.⁶ Third, it replicates the full functionality of the latest 'standard' GTAP model including an interface to the revised database.

Though it is designed to replicate the GEMPACK version of the GTAP model, it also contains a few extensions. Among these are:

- 1) Aggregate factor supply is explicitly modeled with the possibility of an upward sloping supply curve.
- 2) Upward sloping supply curves for natural resources.
- 3) Allocation of domestic output across destination markets is specified using a nested constant-elasticity-of-transformation (CET) structure, analo-

²Complete documentation for GEMPACK is available at <https://www.copsmodels.com/gempack.htm>.

³Complete documentation for GAMS is available at <https://www.gams.com/>.

⁴An earlier version of the GAMS version of the GTAP model reflects version 6.2a. The version described herein only has minor modifications as the new GEMPACK version of the GTAP model incorporates many features from the earlier GAMS implementation. The main changes are: 1) converting the data input module to the new format of GTAP Data Base; and 2) converting the model's price indices to chain-based Fisher 'ideal' price indices.

⁵There are of course many other GAMS-based models that rely on the GTAP Data Base. A partial list can be found at www.gtap.agecon.purdue.edu/about/data_models.asp.

⁶Note that Rutherford supplies two versions of his model—a GAMS-based Mixed Complementarity Problem (MCP) formulation and a Mathematical Programming System for General Equilibrium (MPSGE) version (that is also coupled with GAMS). The version described herein adheres to the GAMS/MCP formulation.

gously to the constant-elasticity-of-substitution (CES) specification of the allocation of domestic absorption. The specification allows for perfect transformation, the explicit assumption in the GTAP model.

- 4) To simplify coding, there is a single set of Armington agents that includes firms, private, public expenditures, and investment expenditures and supply of international trade and transport margins.⁷ The top-level Armington decomposition across agents then only needs a single module.⁸
- 5) The top level Armington substitution elasticity is indexed by Armington agent—though in practice is typically uniform.
- 6) The model includes additional options for the capital account closure. The standard options include capital flows responding to relative changes in regional rates of returns and a fixed allocation of global investment across regions. A third option fixes capital flows in level terms, which given the structure of the balance of payments, fixes the trade account. A fourth option fixes the capital account relative to regional income. This option requires a residual region to absorb slack in the global capital account.

This article is not intended as an introduction to the GTAP model. Potential users are strongly urged to consult [Corong et al. \(2017\)](#). Other key references, particularly to the 'classic' version of the GTAP model include [Hertel \(1997\)](#), especially Chapter 2, [McDougall \(2003\)](#) for a description of the representative household module, [Brockmeier \(2001\)](#) for a graphical description of the GTAP model and [McDonald and Thierfelder \(2004\)](#) for a description of the Social Accounting Matrix (SAM) structure of the underlying database. The description of the separate modules of the model provide some guide to the linkages between the GAMS and GEMPACK nomenclature. The GAMS code provides additional references to the links across model specifications regarding the equation nomenclature—and the equations from both models are linked formally in [Table A.1](#). The GAMS code does not make specific reference to the equation numbers herein, but the sequence of the equations is identical in both this document and the GAMS code to facilitate navigation.

The next section provides a snapshot of the GTAP model, i.e. its main features. That is followed by a full and detailed description of each module of the model. The main approach is that of the circular flow of the economy. It starts with production that generates factor income. Income is distributed across different agents, for example private and public expenditures and savings. The demand modules follow, describing demand at the so-called Armington level. This is followed by the trade section—the allocation of Armington demand across different regions of

⁷[Armington \(1969\)](#) in a seminal paper described import demand using a differentiated goods model. His specification is applied to each agent in the economy.

⁸This simplification would also extend to a multi-regional input-output (MRIO) version of the model.

origin, and the allocation of domestic supply across different regions of destination. There is a short section describing the demand and supply of international trade and transport services. Equilibrium on the goods and factor markets are subsequently described. And a section on investment behavior and closure finishes the description of the static model. A final module describes some potential dynamic elements of the model including changes in technology and preferences. We conclude the description of the model with a summary description of the price relations—a key to understanding the channels through which end-user prices change—and a brief section on the differences between the GAMS and GEMPACK implementations (section 3.22).

A subsequent section describes the accounting framework of the model. Two accounting frameworks will be described. The first is an analytical SAM as derived from model results. The second describes the correspondence between the GTAP Data Base and the variables from the GAMS model. The depiction of the SAM does not represent the full functionality of the underlying database or model. For example, demand is specified at the Armington level. This is to keep the size of the SAM at a reasonable level and yet illustrate the main accounting relations.

A final section highlights numerically four of the model extensions: the CET specification for allocating domestic production by destination market, the fixed capital account closure, and positive supply elasticities for land and natural resources. The simulations use a relatively stylized version of the GTAP Data Base and are meant to illustrate the model extensions as opposed to providing findings from detailed policy analysis.

2. The standard GTAP model in a nutshell

The standard GTAP model is a fairly straightforward comparative static global computable general equilibrium model.⁹ It is multi-sectoral—with up to 57 sectors¹⁰—and multi-region—with up to 140 regions.¹¹ Each regional¹² module has an identical specification (i.e. the GTAP model uses a template model for each region), though with region specific parameterization that depends on the underlying database (largely an input/output table) and region-specific key parameters such as supply and demand elasticities. The regional models are linked through two sets of relations. The first set is trade and the GTAP model traces the flow of bilateral trade between any pair of regions for each sector. Each bilateral trade node is also identified with four sets of prices—the supply price of exports, the border price of exports (or the Free on Board (FOB) price) that incorporates export

⁹Section 2 of [Corong et al. \(2017\)](#) provides a useful discussion about the design philosophy of the GTAP model.

¹⁰Through version 9 of the GTAP Data Base release.

¹¹Through version 9 of the GTAP Data Base release.

¹²The term region is used generically—in many cases a region may refer to a single country.

taxes and subsidies, the border price of imports (or the Cost, Insurance and Freight (CIF) price) that incorporates international trade and transport margins and the domestic post-border price of imports that incorporates tariff measures (but excludes domestic end-user taxes on imports). The GTAP model also allows for endogenous international capital flows, which respond to relative changes in anticipated rates of returns to capital across regions.¹³

The production side of each regional model is based on a nested structure of CES functions to represent the key substitution across inputs—both intermediate goods and factors of production. There is a single CES nest for factor inputs, or endowments, which includes capital, land (in agriculture), a natural resource endowment (in some sectors such as fossil fuels and forestry) and labor, of which there can be several types.¹⁴

The model differentiates between production activities and purchased commodities. This allows for joint production, e.g. a sugar-based ethanol sector can produce ethanol, sugar, rum and bagasse. It also permits a purchased commodity to be produced by multiple activities—electricity providing a natural example. Though the default GTAP Data Base has a diagonal ‘make’ matrix, through aggregation, or a supplemental database, the model allows for a non-diagonal make matrix.

Factor income and revenues generated by taxes are all allocated to a single representative household for each region. The representative household is endowed with a nested structure of preference functions to allocate regional income between demand for goods and services and savings. The specification of top nest uses a Cobb-Douglas preference function that allocates income between private consumption, public (or government) consumption, and regional savings. Private consumption is allocated across goods and services using a constant-difference-in-elasticity (CDE) utility function that is non-homothetic and allows for relatively flexible price response. Government expenditure is allocated across goods and services using a CES preference function (with a default CES elasticity of 1). Regional investment expenditures are similarly allocated using a CES preference function (with a default CES elasticity of 0). Regional investment is equal to regional savings adjusted by international capital flows further described below.

Each agent’s demand for goods and services is first specified at the so-called Armington level, i.e. a composite commodity that includes both domestically produced and imported goods. Each agent then decomposes the demand for the composite bundle into demand for a domestically produced good and an (aggregate) imported good using a CES preference function. The sum across agents of the former is then equated to domestic supply of domestic goods to derive the equilib-

¹³The standard GTAP model includes a second closure for the allocation of global savings that ignores changes in relative rates of return. The GAMS version includes two additional capital account closures. All are described below.

¹⁴Version 9 of the GTAP Data Base increased the number of labor types from 2 (unskilled and skilled) to 5 using data from the International Labour Organization (ILO).

rium price of domestically produced goods sold to the domestic market. Aggregate import demand needs to be allocated across source regions. In the standard GTAP model this is done by an aggregate import agent. Thus import demand (for the composite import good) is summed across all agents, and this aggregate import demand is allocated across all sourcing regions using a CES preference function.¹⁵

Factor supplies are assumed to be exogenous in the standard GTAP model. The formulation herein allows for upward sloping supply curves, albeit the default elasticities are zero. Most factors are assumed to be partially or fully mobile across production activities—the former are sometimes referred to as sluggish factors. The degree of mobility is captured with a CET supply function. The supply of natural resources is assumed to be sector specific, with potentially positive supply elasticities. International capital flows are captured with a construct called the *global bank*. The latter collects savings across all regions and re-allocates these savings in a way that captures relative changes in the expected rates of returns across regions. This is more fully described below.

Like most CGE models, GTAP incorporates a wide range of price wedges—mostly in the form of taxes and subsidies. All of these are fully detailed below. As well, the GTAP model incorporates a wide range of levers for technology and preference changes—suited for both comparative static and dynamic analysis.

3. Model specification in GAMS

3.1 Set definitions

This section describes the principal sets used in the description of the model and also correspond to those used in the GAMS code. Table 1 provides the key indices and sets using the model description. The following are a few additional notes in reference to the table:

- 1) The code differentiates between activities (i.e. production), indexed by a , and commodities indexed by i .
- 2) The regional supplier of international trade and transport margins is assumed to be an Armington agent. This is an extension of the standard GTAP model, however, with no impact as the Armington demand is *de facto* equated to domestically sourced goods. This specification is largely for convenience as it allows for collapsing many of the final demand equations using a generic specification.
- 3) Almost all model equations are indexed by r which is the regional dimension. Bilateral variables need two regional indices. The first is always the

¹⁵There is considerable ongoing research to make the second CES nest be agent specific, i.e. each agent in the economy would source by region directly. This literature is referred to as multi-regional input-output or MRIO. Carrico (2017) describes a GTAP-based MRIO database and implementation of an MRIO specification in the standard GTAP model.

region of origin, or source region. The second is always the destination region. For transparency, the regional index s is used in describing the import demand specification, to reflect the *source* or exporting region. The regional index d is used in the export supply specification to reflect the *destination* or importing region.

- 4) The allocation of factors of production relies on the segmentation of factors that are partially or fully mobile fm and those that are sector-specific fsm . The GAMS version of the code does not differentiate between sluggish and perfectly mobile factors. This is driven instead by the input parameters. If the transformation elasticity is ∞ , then the model will ensure that the law-of-one-price holds and allows for perfect factor mobility.¹⁶

In addition to the key indices identifying sectors, regions and agents, all model equations and variables are indexed by an index ' t ', or time. In comparative static versions of the model, this index is used to identify different shocks. For example, it might be defined as:

```
set t "Time framework"  
    / base, check, shock1, shock2, ... shockn / ;
```

The 'period' `base` could reflect the initial initialization of the model. The 'period' `check` would be an initial simulation of the model with no shocks—in principle, reproducing the base equilibrium. The remaining 'periods' would be specific shocks such as a change to a tax or a productivity parameter. In a dynamic setting, the time framework would reflect years, such as `set t / 2011*2030 / ;`.

3.2 Production

Production is implemented as a series of nested CES functions. The main objective is to capture the key substitution and complementary relations across the various inputs. In the standard GTAP Data Base the inputs include intermediate goods and services, indexed by i for each activity a , and factors of production, indexed by f for each activity a . The production nest is depicted graphically in figure 1.

The top CES nest consists of two bundles. The first composite bundle is an aggregate of the factors of production, i.e. the value added bundle (VA). The second bundle is an aggregate of intermediate demand (ND). Equations (1) and (2) represent the CES derived demand functions for the two bundles, respectively VA and ND . The bundle prices are PVA and PND respectively, where PX represents the aggregate price of the two bundles. Given the assumption of constant-returns-to-scale the price PX is also the unit cost of production. The parameters α^{va} and α^{nd} are

¹⁶GAMS will interpret the value `INF` as infinity and allows limited operations with this value—in particular testing in logical expressions.

Table 1. Sets used in model specification.

Set	Alias	Description
<i>i</i>		Commodities
<i>a</i>		Activities
<i>h</i>		Household (or private) agent
<i>gov</i>		Government (or public) agent
<i>inv</i>		Investment agent
<i>tmg</i>		Domestic supplier of margins
<i>aa</i>		Armington agents = $\cup \{a, h, gov, inv, tmg\}$
<i>fd(aa)</i>		Final demand accounts = $\cup \{h, gov, inv, tmg\}$
<i>m(i)</i>		International trade and transport services
<i>f</i>		Factors of production
<i>l(f)</i>		Labor types (e.g. unskilled and skilled)
<i>cap(f)</i>		Capital factor
<i>lnd(f)</i>		Land factor
<i>nr(f)</i>		Natural resource factor
<i>fm(f)</i>		Partially or fully mobile factors
<i>fnm(f)</i>		Sector-specific factors
<i>r</i>	<i>s, d</i>	Regions of the model
<i>HIC(r)</i>		High-income regions
<i>MANU(i)</i>		Manufactured commodities

Source: Author.

the standard CES (dual) share parameters. There are three technology coefficients. A^{xp} represents a uniform shifter, whereas λ^{va} and λ^{nd} are bundle specific shifters. The CES substitution elasticity is given by σ^p .¹⁷

$$VA_{r,a} = \alpha_{r,a}^{va} X P_{r,a} \left(\frac{PX_{r,a}}{PVA_{r,a}} \right)^{\sigma_{r,a}^p} \left(A_{r,a}^{xp} \lambda_{r,a}^{va} \right)^{\sigma_{r,a}^p - 1} \quad (1)$$

$$ND_{r,a} = \alpha_{r,a}^{nd} X P_{r,a} \left(\frac{PX_{r,a}}{PND_{r,a}} \right)^{\sigma_{r,a}^p} \left(A_{r,a}^{xp} \lambda_{r,a}^{nd} \right)^{\sigma_{r,a}^p - 1} \quad (2)$$

Equation (3) represents the unit cost of production, PX , and is formulated using the CES dual price expression.¹⁸

$$PX_{r,a} = \frac{1}{A_{r,a}^{xp}} \left[\alpha_{r,a}^{va} \left(\frac{PVA_{r,a}}{\lambda_{r,a}^{va}} \right)^{1-\sigma_{r,a}^p} + \alpha_{r,a}^{nd} \left(\frac{PND_{r,a}}{\lambda_{r,a}^{nd}} \right)^{1-\sigma_{r,a}^p} \right]^{\frac{1}{1-\sigma_{r,a}^p}} \quad (3)$$

¹⁷The substitution elasticity in GTAP is given by ESUBT.

¹⁸There is equivalence between the CES dual price formula and the zero profit condition that is expressed as:

$$PX_{r,a} X P_{r,a} = PVA_{r,a} VA_{r,a} + PND_{r,a} ND_{r,a}$$

Herein the dual price expression will be used at all times.

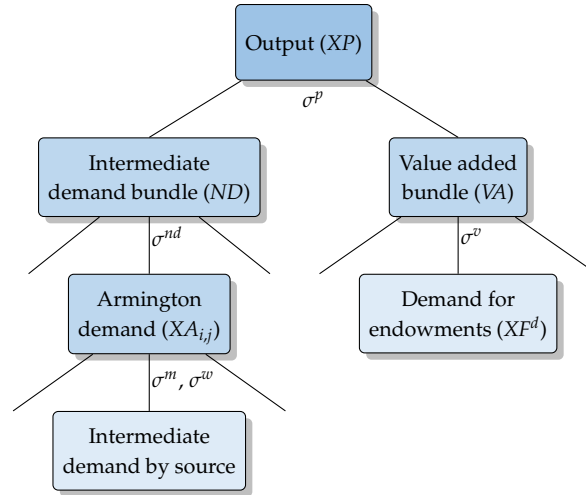


Figure 1. Production CES nest

This subunit determines VA , ND and PX that correspond to variables q_{va} , q_{int} and p_o in the TABLO version¹⁹ of the GTAP model.

The following nest decomposes the aggregate value added bundle, VA , into its components, i.e. the various factors of production. A single nest is used in the standard GTAP model, though many of the GTAP variants allow for a more complex nesting—for example the isolation of the land factor in the agricultural sectors. Equation (4) determines the derived demand for factor f ,²⁰ XF^d , where the CES substitution elasticity across factors is given by σ^v .²¹ The purchasers' (or agents') price of factors is given by PF^a . The parameter α^f represents the standard CES (dual) share parameter. Factor specific technology shifters are given by λ^f . Equation (5) determines the aggregate value added price, PVA , using the CES dual price aggregation expression.

$$XF_{r,f,a}^d = \alpha_{r,f,a}^f VA_{r,a} \left(\frac{PVA_{r,a}}{PF_{r,f,a}^a} \right)^{\sigma_{r,a}^v} \left(\lambda_{r,f,a}^f \right)^{\sigma_{r,a}^v - 1} \quad (4)$$

$$PVA_{r,a} = \left[\sum_f \alpha_{r,f,a}^f \left(\frac{PF_{r,f,a}^a}{\lambda_{r,f,a}^f} \right)^{1 - \sigma_{r,a}^v} \right]^{1 / (1 - \sigma_{r,a}^v)} \quad (5)$$

¹⁹Note that most of the variables in the GEMPACK version of the GTAP model represent percent changes of the relevant variable—where the use of lower case variable names is typically understood to represent percent changes. Upper case names typically represent so-called coefficients and are either initialized at the beginning of a model simulation or updated between solution iterations.

²⁰Factors are indexed by e in the GEMPACK code.

²¹The substitution elasticity in GTAP is given by $ESUBVA$.

This subunit determines XF^d and PVA corresponding to variables qfe and pva in the GEMPACK code.

The next set of equations relate to the intermediate demand nest of the model. It decomposes the ND bundle into demand for goods and services by sector. Equation (6) determines intermediate demand by commodity (at the Armington level), XA , with (Armington) prices given by PA .²² The substitution elasticity is given by σ^{nd} and is typically 0.²³ Technology shifters are provided by the λ^{io} coefficients. Equation (7) determines the price of the aggregate intermediate demand bundle PND using the CES dual price expression.

$$XA_{r,i,a} = \alpha_{r,i,a}^{io} ND_{r,a} \left(\frac{PND_{r,a}}{PA_{r,i,a}} \right)^{\sigma_{r,a}^{nd}} \left(\lambda_{r,i,a}^{io} \right)^{\sigma_{r,a}^{nd}-1} \quad (6)$$

$$PND_{r,a} = \left[\sum_i \alpha_{r,i,a}^{io} \left(\frac{PA_{r,i,a}}{\lambda_{r,i,a}^{io}} \right)^{1-\sigma_{r,a}^{nd}} \right]^{1/(1-\sigma_{r,a}^{nd})} \quad (7)$$

This subunit determines $XA_{i,a}$ and PND . These correspond to variables qfa and $pint$ in the GTAP code.

The decomposition of the Armington demand will be discussed below in the trade section. It is consolidated for all Armington agents, unlike the GEMPACK code that has it specified separately for each Armington agent.

3.3 Commodity supply

The GTAP model distinguishes between activities and commodities. A 'make' matrix is used to convert output of activity a into one or more commodities indexed by i . In other words, activities can produce one or more commodities, for example the ethanol sector could produce both ethanol and distiller's dried grains with solubles (DDGS)—a valuable feed for livestock. A CET specification is used to allocate the production of activity a into supply of its various commodities. Similarly, a national agent buys various commodities labeled i produced by one or more activities to provide a national, or aggregate, supply of good i . For example a national electricity supplier could buy electricity from different power generators—thermal, nuclear, hydro, renewables, etc. A CES specification is used to aggregate output from one or more activities.

²²The Armington variables and all of their components are indexed by i , i.e. commodity, and aa that indexes all of the Armington agents. The Armington agents include all firms (or activities) indexed by a , households indexed by h , the government sector indexed by gov , the investment sector indexed by inv and trade margins, indexed by tmg . The latter is for convenience only as the data assumes that all exported international trade and transport services are sourced within the exporting region.

²³The substitution elasticity in GTAP is given by $ESUBC$.

3.3.1 Commodity supply

Equation (8) represents the supply of commodity i produced by activity a , $X_{a,i}$, derived from a standard CET specification, where ω^s represents the transformation elasticity and γ^x are the standard CET share parameters. In the classic GTAP model, the matrix X is diagonal, i.e. each activity produces one and only one commodity.²⁴ In this formulation, each activity can produce one or more commodities. If the commodities being produced are homogeneous, i.e. the transformation elasticity given by ω^s is infinite, the law-of-one-price holds. Equation (9) represents the zero profit condition as expressed in the dual price expression or the aggregation of commodities in the case of perfect transformation, i.e. commodity homogeneity.²⁵

$$\begin{cases} X_{r,a,i} = \gamma_{r,a,i}^x X P_{r,a} \left(\frac{P_{r,a,i}}{P X_{r,a}} \right)^{\omega_{r,a}^s} & \text{if } \omega_{r,a}^s \neq \infty \\ P_{r,a,i} = P X_{r,a} & \text{if } \omega_{r,a}^s = \infty \end{cases} \quad (8)$$

$$\begin{cases} P X_{r,a} = \left[\sum_i \gamma_{r,a,i}^x P_{r,a,i}^{1+\omega_{r,a}^s} \right]^{1/(1+\omega_{r,a}^s)} & \text{if } \omega_{r,a}^s \neq \infty \\ X P_{r,a} = \sum_i X_{r,a,i} & \text{if } \omega_{r,a}^s = \infty \end{cases} \quad (9)$$

At this stage, a price wedge is introduced that represents a tax on output. It is indexed by activity of origin and commodity and is applied to price P . Equation (10) defines the supply price of commodity i produced by activity a , PP , where τ^p represents the output tax.

$$PP_{r,a,i} = P_{r,a,i} \left(1 + \tau_{r,a,i}^p \right) \quad (10)$$

3.3.2 Commodity demand

In an analogous fashion, equation (11) represents the demand side of the 'make' matrix. A national buyer of commodity i purchases goods from the different national producers (indexed by a) using a CES preference function. National supply of commodity i is represented by the variable XS . It eventually will be allocated to domestic and export markets (see below). The specification allows for commodity homogeneity if the substitution elasticity, σ^s , is infinite. Equation (12) represents

²⁴Though the classic GTAP Data Base has a diagonal 'make' matrix, it is possible to make it non-diagonal by providing two separate mappings for produced and consumed commodities. For example, one could aggregate all agricultural production into a single activity, but have it produce a variety of agricultural commodities.

²⁵The transformation elasticity ω^s is represented by the parameter `ETRAQ` in the GEMPACK model.

the standard zero-profit condition for both the finite and infinite cases.²⁶

$$\begin{cases} X_{r,a,i} = \alpha_{r,a,i}^x X S_{r,i} \left(\frac{P S_{r,i}}{P P_{r,a,i}} \right)^{\sigma_{r,i}^s} & \text{if } \sigma_{r,i}^s \neq \infty \\ P P_{r,a,i} = P S_{r,i} & \text{if } \sigma_{r,i}^s = \infty \end{cases} \quad (11)$$

$$\begin{cases} P S_{r,i} = \left[\sum_a \alpha_{r,a,i}^x P P_{r,a,i}^{1-\sigma_{r,i}^s} \right]^{1/(1-\sigma_{r,i}^s)} & \text{if } \sigma_{r,i}^s \neq \infty \\ X S_{r,i} = \sum_a X_{r,a,i} & \text{if } \sigma_{r,i}^s = \infty \end{cases} \quad (12)$$

This module determines P , XP , PP , X and PS . The corresponding GEMPACK variables are ps , qo , pca , qca and pds .²⁷

3.4 Income allocation

There are two main sources of income. The first is tax revenues generated by the myriad of taxes and subsidies. The second is the revenues generated by the use of factors in the production of goods. Income is then allocated between private and public expenditure and savings.

Tax revenues are generated by taxes on production inputs (both intermediate goods and factors of production), output taxes, sales taxes on both domestic and import consumption, import and export taxes, and direct taxes on factor income. Equation (13) defines revenues from output taxes. Equation (14) defines tax revenues generated by taxes on commodity sales across all activities. Commodity taxes are allowed to differ between goods sourced domestically and imported. Equations (15), (16) and (17) represent sales taxes on private, public and investment expenditures. Equations (18) and (19) define tax revenues generated by taxes and subsidies on factors of production. The price PF represents the market (or equilibrium) price of factors. Equation (20) defines taxes on imports, where τ^m represents the bilateral tariff rates applied to imports from s into r and PM^{CIF} is the border or CIF price of imports. The variable XW represents bilateral trade flows. The first regional index is the exporting region and the second regional index is the importing region. The index s will be used to indicate the *source* region when used to describe imports and the index d will be used to indicate the *destination* region

²⁶The substitution elasticity σ^s is represented by the parameter $1/ESUBQ$ in the GEMPACK model. The GEMPACK model inverts the standard CES formulation and represents perfect substitutability by setting $ESUBQ$ equal to zero. Note that in the GEMPACK formulation, an elasticity of zero is not allowed, though unlikely to be relevant in most contexts.

²⁷There is a subtle sleight of hand in this formulation. In principle, equation (8) determines the *supply* of X^s , and equation (11) determines the *demand* for X^d . An additional equilibrium equation would set the two separately identified variables to equality and thus determine the market equilibrium price, P . The equilibrium condition is substituted out of the model specification.

when used to describe exports. Equation (21) defines taxes/subsidies on exports, where τ^e represents the bilateral tax/subsidy applied to exports from r imported by d . The tax/subsidy is applied to the producer price of exports, PE . Equation (22) represents the tax revenues generated by direct taxes on factor income where κ^f is the tax rate on income generated by factor f in activity a .

$$YTAX_{r,pt} = \sum_a \sum_i \tau_{r,a,i}^p P_{r,a,i} X_{r,a,i} \quad (13)$$

$$YTAX_{r,fc} = \sum_a \sum_i \left[\tau_{r,i,a}^{dtx} PD_{r,i} XD_{r,i,a} + \tau_{r,i,a}^{mtx} PMT_{r,i} XM_{r,i,a} \right] \quad (14)$$

$$YTAX_{r,pc} = \sum_h \sum_i \left[\tau_{r,i,h}^{dtx} PD_{r,i} XD_{r,i,h} + \tau_{r,i,h}^{mtx} PMT_{r,i} XM_{r,i,h} \right] \quad (15)$$

$$YTAX_{r,gc} = \sum_i \left[\tau_{r,i,gov}^{dtx} PD_{r,i} XD_{r,i,gov} + \tau_{r,i,gov}^{mtx} PMT_{r,i} XM_{r,i,gov} \right] \quad (16)$$

$$YTAX_{r,ic} = \sum_i \left[\tau_{r,i,inv}^{dtx} PD_{r,i} XD_{r,i,inv} + \tau_{r,i,inv}^{mtx} PMT_{r,i} XM_{r,i,inv} \right] \quad (17)$$

$$YTAX_{r,ft} = \sum_a \sum_f \left[\tau_{r,f,a}^{ft} PF_{r,f,a} XF_{r,f,a}^d \right] \quad (18)$$

$$YTAX_{r,fs} = \sum_a \sum_f \left[\tau_{r,f,a}^{fs} PF_{r,f,a} XF_{r,f,a}^d \right] \quad (19)$$

$$YTAX_{r,mt} = \sum_i \sum_s \left[\tau_{s,i,r}^m PM_{s,i,r}^{CIF} XW_{s,i,r} \right] \quad (20)$$

$$YTAX_{r,et} = \sum_i \sum_d \left[\tau_{r,i,d}^e PE_{r,i,d} XW_{r,i,d} \right] \quad (21)$$

$$YTAX_{r,dt} = \sum_f \left[\sum_a \kappa_{r,f}^f PF_{r,f,a} XF_{r,f,a} \right] \quad (22)$$

This subunit generates $YTAX$ with its various components. In the GEMPACK version the corresponding variables represent the (ordinary) change in the respective revenue stream and not the percent change. These variables are `del_taxrout` (output tax), `del_taxrgc` (tax on government consumption), `del_taxrpc` (tax on private consumption), `del_taxriu` (tax on intermediate consumption and investment goods), `del_taxrfu` (tax on factor inputs), `del_taxrimp` (tax on imports), `del_taxrexp` (tax on exports) and `del_taxrinc` (direct taxes on factor

income).²⁸

Total revenues from taxes is defined in equation (23), where the sum is over all tax revenue streams as defined by the index gy . Total revenues from indirect taxes is equal to total taxes less direct taxes, equation (24).

$$YTaxTot_r = \sum_{gy} YTAX_{r,gy} \quad (23)$$

$$YTaxInd_r = YTaxTot_r - YTAX_{r,dt} \quad (24)$$

This subunit calculates $YTaxTot$ and $YTaxInd$. These variables correspond to the GEMPACK version variables `del_ttaxr` and `del_indtaxr`.

Equation (25) represents total factor income net of depreciation. Note that factor income is defined at market prices, not net of direct taxes. The variable PI represents the unit cost of investing, i.e. the replacement cost of capital goods, K^0 is the beginning of period capital stock and δ is the depreciation rate. Total regional income, Y , is defined in equation (26) and is equal to the sum of factor income net of depreciation and total revenues generated by indirect taxes.

$$factY_r = \sum_f \sum_a [PF_{r,f,a} XF_{r,f,a}] - \delta_r PI_r K_r^0 \quad (25)$$

$$Y_r = factY_r + YTaxInd_r \quad (26)$$

This subunit calculates $factY$ and Y corresponding to the GEMPACK variables `fincome` and `y`.

3.5 Allocation of regional income

There are three domestic final demand agents—private households, government and investment. Their demand is specified with a nested preference structure that first allocates total regional income to the three agents and then each agent has an agent-specific preference function that determines the demand for goods and services. The nested demand structure is depicted in Figure 2.²⁹

Regional income, Y , is allocated across three agents using a top level Cobb-Douglas utility function. The three aggregate expenditure categories are private and public expenditures and aggregate savings. The representative regional household is assumed to maximize utility according to the following scheme:

²⁸The standard GTAP model does not make use of factor subsidies (data contained in the array `FBEP` in the GTAP Data Base).

²⁹The graphic depicts a situation excluding international capital flows. The top level Cobb-Douglas preference structure determines the level of domestic savings. The level of domestic investment will differ to the extent the region attracts savings from abroad, or vice-versa.

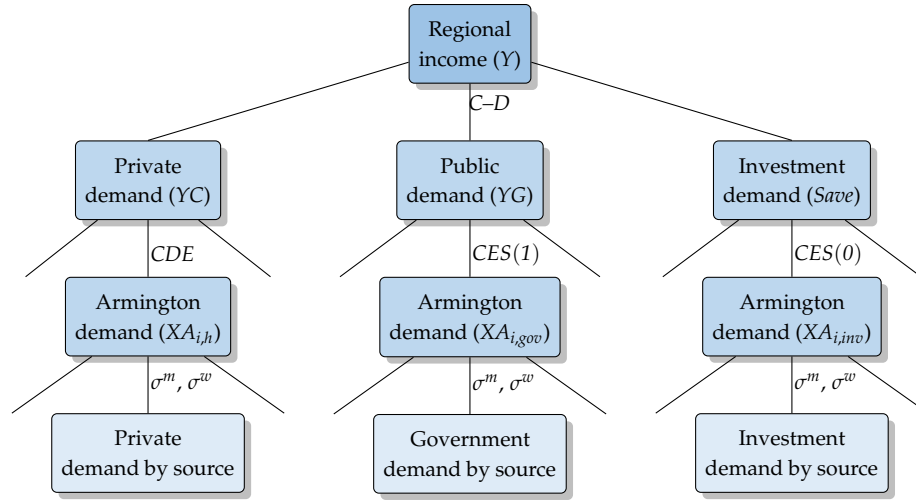


Figure 2. Demand nest

$$\max U_r = A_r^u U_r^p \beta_r^p U_r^G \beta_r^G U_r^S \beta_r^S$$

subject to

$$Y_r = E_r^P(U_r^P, P_r^P) + E_r^G(U_r^G, P_r^G) + E_r^S(U_r^S, P_r^S)$$

where the superscript indices refer respectively to private or household consumption (P), government or public consumption (G) and savings (S). The expenditure functions for government and savings are both of a generic CES variety and thus the expenditure function can be written as $E(u, P) = A \cdot u \cdot f(P)$, where $f(P)$ in this case is the CES dual price expression. The household expenditure function is based on a CDE utility function and has no simple expression. The derivation of the expenditure shares requires expressions for the elasticity of total expenditure with respect to total utility, Φ , that in turn requires the elasticity of private expenditure with respect to private utility, (see McDougall (2003)). Equation (27) determines the latter. For the CDE expenditure function, the elasticity of expenditure with respect to utility is the weighted sum of the CDE expansion parameters (e), where the weights are given by the (private consumption) budget shares (s_i^p).³⁰ The total expenditure elasticity (with respect to total utility) is given in equation (28) and is the inverse of the sum of the individual Cobb-Douglas share parameters with weights given by the inverse of the individual expenditure elasticities.

³⁰The expenditure elasticity for the other two expenditure functions is 1 due to the form of the expenditure function, i.e. the use of a CES function.

$$\phi_r^P = \sum_i s_{r,i}^p e_{r,i} \quad (27)$$

$$\Phi_r = \left[\beta_r^P / \phi_r^P + \beta_r^G + \beta_r^S \right]^{-1} \quad (28)$$

The expenditures are derived from utility maximization and are given in equations (29) through (31). The equations determine respectively aggregate private consumption (YC), aggregate public consumption (YG) and total regional savings ($Save$). The expenditures are provided at aggregate level since regional income (Y) is aggregate regional income. Per capita levels can be determined by dividing through by population.

$$YC_r = \beta_r^P \frac{\Phi_r}{\phi_r^P} Y_r \quad (29)$$

$$YG_r = \beta_r^G \Phi_r Y_r \quad (30)$$

$$Save_r = \beta_r^S \Phi_r Y_r \quad (31)$$

This subunit derives ϕ^P , Φ , YC , YG , and $Save$. The corresponding variables in the GEMPACK version are `uepriv`, `uelas`, `yp`, `yg` and `qsave`.

3.6 Utility of representative household

The top level utility function depends on the utility of the sub-components. Equation (32) defines (implicitly) utility from private consumption based on the CDE utility function, U^P . It is exclusively a function of consumer prices and per capita private expenditure and the parameters of the utility function. The CDE function is more fully described in Hertel (1997). The e parameters are known as the expansion parameters and are linked to the income elasticities. The b parameters are the substitution parameters and linked to the price elasticities. The share parameters (α^a) and the consumer (Armington) prices are both indexed by h , which is an index in the set of Armington agents. Utility is per capita as aggregate private expenditure is divided by total population. Utility from public expenditure, U^G , and savings, U^S , are provided in equations (33) and (34) respectively, where XG is the aggregate volume of public spending and $Save$ is nominal savings that is divided by the price (index) of savings, $PSave$. These functional forms can be derived from the generic CES expenditure function. Equation (35) defines total (per capita) utility, U .

$$\sum_i \alpha_{r,i,h}^a PA_{r,i,h}^{b_{r,i}} (U_r^P)^{b_{r,i}e_{r,i}} \left(\frac{YC_r}{Pop_r} \right)^{-b_{r,i}} \equiv 1 \quad (32)$$

$$U_r^G = A_r^{ug} \frac{XG_r}{Pop_r} \quad (33)$$

$$U_r^S = A_r^{us} \frac{Save_r / PSave_r}{Pop_r} \quad (34)$$

$$U_r = A_r^u U_r^{P\beta_r^P} U_r^{G\beta_r^G} U_r^{S\beta_r^S} \quad (35)$$

This subunit generates U^P , U^G , U^S and U corresponding to the GEMPACK variables up, ug, UTILSAVE³¹ and u.

3.7 Private consumption

Consumer demand as derived from the CDE utility function is defined by the equation below (which will be simplified subsequently). The ratio defines per capita consumption that is then multiplied by population to derive aggregate private consumption, XA . The latter is part of the Armington matrix that covers all Armington agents.

$$XA_{r,i,h} = Pop_r \frac{\alpha_{r,i,h}^a b_{r,i} PA_{r,i,h}^{(b_{r,i}-1)} (U_r^P)^{b_{r,i}e_{r,i}} \left(\frac{YC_r}{Pop_r} \right)^{(1-b_{r,i})}}{\sum_j \alpha_{r,j,h}^a b_{r,j} PA_{r,j,h}^{b_{r,j}} (U_r^P)^{b_{r,j}e_{r,j}} \left(\frac{YC_r}{Pop_r} \right)^{b_{r,j}}}$$

If we define the following auxiliary variable:

$$Z_{r,i,h} = \alpha_{r,i,h}^a b_{r,i} PA_{r,i,h}^{b_{r,i}} (U_r^P)^{b_{r,i}e_{r,i}} \left(\frac{YC_r}{Pop_r} \right)^{-b_{r,i}}$$

then the expression for the budget shares is given by equation (36).

$$s_{r,i,h}^p = \frac{Z_{r,i,h}}{\sum_j Z_{r,j,h}} \quad (36)$$

³¹There is no direct correspondence with U^S in the GEMPACK code, it is essentially substituted out and the update of the coefficient UTILSAVE is based on the percent change in the variable qsave less the percent change in population.

Moreover, the utility expression, equation (32) simplifies to:

$$\sum_i \frac{Z_{r,i,h}}{b_{r,i}} \equiv 1$$

Given the budget shares, aggregate consumption is readily evaluated using equation (37).

$$XA_{r,i,h} = \frac{s_{r,i,h}^p YC_r}{PA_{r,i,h}} \quad (37)$$

Equation (38) provides one definition of the consumer price index, PC .

$$PC_r = \sum_i s_{r,i}^p PA_{r,i,h} \quad (38)$$

The model also incorporates a variant of the CDE utility function where the expansion parameter, e , is uniformly 1 and the substitution parameter, b , is uniformly 0. This is the familiar Cobb-Douglas utility function.³² It has been incorporated in the model for the purpose of using the *Altertax* procedure where the key elasticities are set (close) to 1 in order to preserve budget shares near their original levels.³³

The equations above need some adjustment. The utility expression for the Cobb-Douglas utility function is:

$$U_r^p = AU_r^p \prod_i XA_{r,i,h}^{\alpha_{r,i,h}^a}$$

And, the auxiliary variable simplifies to:

$$Z_{r,i,h} = \alpha_{r,i,h}^a$$

where the α^a parameters must add up to 1.

This subunit generates the variables $XA_{r,i,h}$, s^p and PC corresponding to the GEMPACK variables `qp` (private consumption) and `ppriv` (consumer price index). The budget shares in GEMPACK (`CONSHR`) are treated as coefficients in the code and are updated at each iteration.

³²The model user selects the choice of utility function by setting the global parameter `utility`. Valid choices are `CDE`, the default, and `CD` for the Cobb-Douglas variant.

³³See [Malcolm \(1998\)](#).

3.8 Government consumption

The allocation of aggregate government expenditure across goods and services uses a CES expenditure function.³⁴ Equation (39) determines the *gov* vector in Armington demand, XA .

$$XA_{r,i,gov} = \alpha_{r,i,gov}^a XG_r \left(\frac{PG_r}{PA_{r,i,gov}} \right)^{\sigma_r^g} \quad (39)$$

The government expenditure price deflator, PG , is defined in equation (40).³⁵

$$PG_r = \left[\sum_i \alpha_{r,i,gov}^a PA_{r,i,gov}^{(1-\sigma_r^g)} \right]^{1/(1-\sigma_r^g)} \quad (40)$$

The volume of government expenditure, XG , is defined in equation (41).

$$YG_r = PG_r XG_r \quad (41)$$

This subunit determines the variables $XA_{r,i,gov}$, PG and XG corresponding to GEMPACK variables *qg* (government purchases of goods and services) and *pgov* (government expenditure price deflator).³⁶

3.9 Investment expenditure

Investment expenditure, similar to government expenditure, is determined using a generic CES expenditure function (with a default substitution elasticity of 0). Equation (42) determines the *inv* vector in Armington demand, XA . The specification allows for technological changes as measured by the variable λ^i .

$$XA_{r,i,inv} = \alpha_{r,i,inv}^a XI_r \left(\lambda_{r,i}^i \right)^{\sigma_r^i - 1} \left(\frac{PI_r}{PA_{r,i,inv}} \right)^{\sigma_r^i} \quad (42)$$

The investment expenditure price deflator, PI , is defined in equation (43).³⁷

$$PI_r = \left[\sum_i \alpha_{r,i,inv}^a \left(\frac{PA_{r,i,inv}}{\lambda_{r,i}^i} \right)^{(1-\sigma_r^i)} \right]^{1/(1-\sigma_r^i)} \quad (43)$$

³⁴The default substitution elasticity is 1, i.e. a Cobb-Douglas expenditure function.

³⁵The code uses the Cobb-Douglas dual price expression if the CES elasticity is 1.

³⁶The GEMPACK code does not explicitly use the aggregate volume of government expenditure, replacing its use by the expression $yg - pgov$ wherever it would be needed, for example equation GOVU.

³⁷The code uses the Cobb-Douglas dual price expression if the CES elasticity is 1.

Investment volume, XI , is defined in equation (44). The nominal level of investment will be determined by the investment closure specification described below.

$$YI_r = PI_r XI_r \quad (44)$$

This subunit determines the variables $XA_{r,i,inv}$, PI and YI corresponding to GEMPACK variables q_{ia} (investment purchases of goods and services) and p_{inv} (investment expenditure price deflator).³⁸

3.10 Top level Armington nest

The standard GTAP model assumes a 2-stage Armington nest, see figure 3. The top level Armington nest is specified at the agents' level that determines the agents' demand for domestic and (aggregate) import goods, respectively. Armington demand for all agents has been described above—for firms, private consumers, the public sector and investment demand.³⁹ At this stage, the Armington demand for each agent (and commodity) is decomposed into a domestic and import component using a CES preference structure.

Agents are faced with market prices given by PD and PMT , respectively for domestic and imported goods. The former represents the market price of domestically produced goods and the latter represents the price of the aggregate import bundle. The agents' prices are equal to the market prices plus an *ad valorem* tax wedge that is agent and commodity specific given by τ^{dtx} and τ^{mtx} respectively for domestic and imported goods. Equations (45) and (46) determine the purchasers' (or agents') prices for domestic and imported goods.

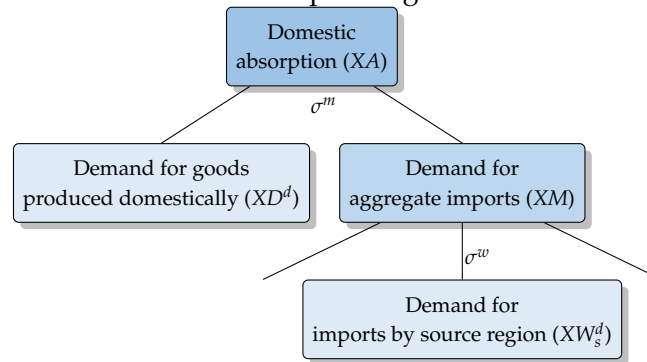


Figure 3. Nested Armington demand

³⁸The GEMPACK code does not explicitly use the nominal value of investment.

³⁹The international trade and transport services sector is also treated as an Armington agent for convenience. The GTAP Data Base and model assume that intermediate demand for this sector is sourced exclusively from the domestic market. The Armington demand for this sector will be described below.

$$PD_{r,i,aa}^p = PD_{r,i} \left(1 + \tau_{r,i,aa}^{dx} \right) \quad (45)$$

$$PM_{r,i,aa}^p = PMT_{r,i} \left(1 + \tau_{r,i,aa}^{mx} \right) \quad (46)$$

The Armington price for each agent, PA , is given by equation (47) that is the CES dual price expression for the aggregate price as a function of the component prices, respectively PD^p and PM^p . The Armington elasticity is given by σ^m .⁴⁰

$$PA_{r,i,aa} = \left[\alpha_{r,i,aa}^d (PD_{r,i,aa}^p)^{1-\sigma_{r,i,aa}^m} + \alpha_{r,i,aa}^m (PM_{r,i,aa}^p)^{1-\sigma_{r,i,aa}^m} \right]^{1/(1-\sigma_{r,i,aa}^m)} \quad (47)$$

The next set of equations, (48) and (49), reflect the Armington decomposition, i.e. the demand for domestic (XD) and imported goods (XM), respectively, for each agent and for each commodity.

$$XD_{r,i,aa} = \alpha_{r,i,aa}^d XA_{r,i,aa} \left(\frac{PA_{r,i,aa}}{PD_{r,i,aa}^p} \right)^{\sigma_{r,i,aa}^m} \quad (48)$$

$$XM_{r,i,aa} = \alpha_{r,i,aa}^m XA_{r,i,aa} \left(\frac{PA_{r,i,aa}}{PM_{r,i,aa}^p} \right)^{\sigma_{r,i,aa}^m} \quad (49)$$

This subunit determines five Armington variables: PD^p , PM^p , PA , XD and XM . The corresponding variables in the GEMPACK code are pfd, pfm, pfa, qfd and qfm for firms, ppd, ppm, ppa, qpd and qpm for private consumption, pgd, pgm, pga, qgd and qgm for public consumption, pid, pim, pia, qid and qim for investment expenditures, and qst for trade and transport margins (where the Armington assumption is used for convenience).⁴¹

3.11 Second level Armington nest

The second level Armington nest decomposes aggregate import demand by region of origin. In principle, this could also be done at the agent level, but for practical reasons, the second level nesting is done at the aggregate regional level, i.e. there is an aggregate importer that allocates aggregate import demand across regions of origin using a CES preference structure. Equation (50) determines aggre-

⁴⁰The top level Armington elasticity in GTAP is given by `ESUBM` that is region- and commodity-specific. In the GAMS version of the model, the Armington elasticity is also agent-specific. In the absence of supplemental data, the parameter will nonetheless be uniform across agents.

⁴¹The price of trade and transport margins is equal to the domestic producer price, i.e. PD in this notation, but `pds` in the GEMPACK notation where no difference is made between domestic and export markets.

gate import demand across all Armington agents, XMT . Equation (51) provides the allocation of aggregate imports across all source regions, indexed by s (that may eventually include the own-region imports if the region is a combination of two or more regions). The variable XW^d represents the demand for exports from region s to region r for commodity i .⁴² The variable PM is the purchasers' price of bilateral imports that is tariff inclusive (to be defined below). The formulation allows for changes in trade preferences as measured by the variable λ^m . The price of aggregate imports, PMT , is defined in equation (52) using the CES dual price expression.

$$XMT_{r,i} = \sum_{aa} XM_{r,i,aa} \quad (50)$$

$$XW_{s,i,r}^d = \alpha_{s,i,r}^{zw} XMT_{r,i} (\lambda_{s,i,r}^m)^{\sigma_{r,i}^w - 1} \left(\frac{PMT_{r,i}}{PM_{s,i,r}} \right)^{\sigma_{r,i}^w} \quad (51)$$

$$PMT_{r,i} = \left[\sum_s \alpha_{s,i,r}^{zw} \left(\frac{PM_{s,i,r}}{\lambda_{s,i,r}^m} \right)^{1 - \sigma_{r,i}^w} \right]^{1 / (1 - \sigma_{r,i}^w)} \quad (52)$$

This subunit generates the variables XMT , XW^d and PMT . The corresponding GEMPACK variables are qms , qxs and pms .

3.12 Allocation of domestic supply

Domestic supply, XS , is sold to the domestic (i.e. regional market) and abroad to the various regions of the model in the form of exports (including own-exports).⁴³ In the standard GTAP model, all output is sold at a uniform producer price that is PS in the GAMS version of the model, and pds in the GEMPACK version. The GAMS version of the model allows for imperfect transformation of domestic supply across various markets of destination, see figure (4). Analogously to the implementation of import demand, the allocation of domestic supply uses a nested CET structure. In the first nest, the aggregate domestic supplier allocates supply between the domestic market and an aggregate exporter. The latter in turn allocates aggregate exports across the various regions of the model thereby determining bilateral export supply.

Equations (53) and (54) describe the top level CET supply functions for the domestic market (XD^s) and aggregate exports (XET), respectively. The formulation allows for the possibility of perfect transformation, which is the explicit assump-

⁴²The superscript d represents the demand for bilateral trade flows. In the model implementation, the trade equilibrium condition is substituted out.

⁴³Recall that in the case of a diagonal 'make' matrix, $XS = XP$ and $PS = PP$.

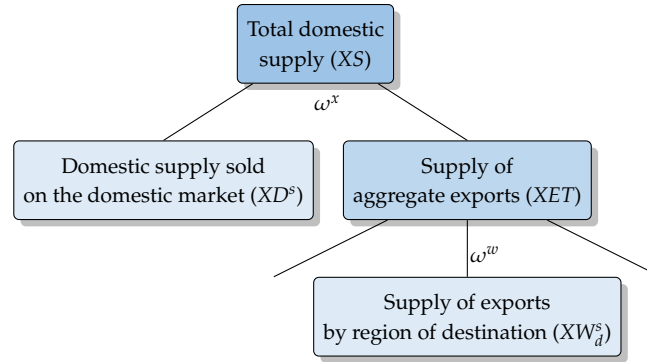


Figure 4. Nested CET transformation of domestic output

tion in the GEMPACK version of the GTAP model. The price PD represents the market price of domestically produced goods sold on the domestic (i.e. regional market). In the case of perfect transformation it must equal the aggregate supply price, i.e. the law-of-one-price holds. The price PET represents the price of aggregate exports, which must also equal the aggregate supply price in the case of perfect transformation. Equation (55) represents the "market clearing" condition for domestic supply. This is clearly the case with perfect transformation. In the case of imperfect transformation, it simply reflects the aggregation condition for domestic supply and its equivalent representation using the CET dual price expression.

$$\begin{cases} XD_{r,i}^s = \gamma_{r,i}^d XS_{r,i} \left(\frac{PD_{r,i}}{PS_{r,i}} \right)^{\omega_{r,i}^x} & \text{if } \omega_{r,i}^x \neq \infty \\ PD_{r,i} = PS_{r,i} & \text{if } \omega_{r,i}^x = \infty \end{cases} \quad (53)$$

$$\begin{cases} XET_{r,i} = \gamma_{r,i}^e XS_{r,i} \left(\frac{PET_{r,i}}{PS_{r,i}} \right)^{\omega_{r,i}^x} & \text{if } \omega_{r,i}^x \neq \infty \\ PET_{r,i} = PS_{r,i} & \text{if } \omega_{r,i}^x = \infty \end{cases} \quad (54)$$

$$\begin{cases} PS_{r,i} = \left[\gamma_{r,i}^d PD_{r,i}^{1+\omega_{r,i}^x} + \gamma_{r,i}^e PET_{r,i}^{1+\omega_{r,i}^x} \right]^{1/(1+\omega_{r,i}^x)} & \text{if } \omega_{r,i}^x \neq \infty \\ XS_{r,i} = XD_{r,i}^s + XET_{r,i} & \text{if } \omega_{r,i}^x = \infty \end{cases} \quad (55)$$

The second level CET nests allocates aggregate exports XET across the various destination export markets and hence defines bilateral export supply. Equation (56) represents the CET supply function in the case of imperfect transformation across export markets, XW^s , that represents the exports from region r to region d for commodity i . In the case of perfect transformation, the supply function is replaced with the law-of-one-price, where the producer price of exports across all regions of destination, PE , is set equal to the producer price of domestic output. Equation (57) represents the CET aggregation function and essentially determines the price of

aggregate exports PET .

$$\begin{cases} XW_{r,i,d}^s = \gamma_{r,i,d}^w XET_{r,i} \left(\frac{PE_{r,i,d}}{PET_{r,i}} \right)^{\omega_{r,i}^w} & \text{if } \omega_{r,i}^w \neq \infty \\ PE_{r,i,d} = PET_{r,i} & \text{if } \omega_{r,i}^w = \infty \end{cases} \quad (56)$$

$$\begin{cases} PET_{r,i} = \left[\sum_d \gamma_{r,i,d}^w PE_{r,i,d}^{1+\omega_{r,i}^w} \right]^{1/(1+\omega_{r,i}^w)} & \text{if } \omega_{r,i}^w \neq \infty \\ XET_{r,i} = \sum_d XW_{r,i,d}^s & \text{if } \omega_{r,i}^w = \infty \end{cases} \quad (57)$$

In the case of perfect transformation at both levels, the equilibrium condition can be replaced by the following expression, which is equivalent to the market clearing condition in the GEMPACK code (E_qds).

$$XS_{r,i} = XD_{r,i}^s + \sum_d XW_{r,i,d}^s$$

This subunit determines the variables XD^s , XET , XS , XW^s and PET corresponding to only the GEMPACK variable qds (output) as the other variables relate to the CET implementation.

3.13 International trade and transport margins

The GTAP Data Base incorporates a wedge between the FOB and CIF price of goods, i.e. the border price of exports and imports. The wedge represents a trade and transport margin. These margins generate a demand for trade and transport services. The "global" trade and transport services sector purchases these services from various source regions using a CES expenditure function. The first set of equations deals with the demand for international trade and transport services. Equation (58) determines the total demand for each of the transportation nodes, $XWMG$, with a simple Leontief assumption. The second equation, (59), breaks out total demand for international trade and transport services by node into different modes indexed by m .⁴⁴ The second level nest also uses a Leontief specification with the additional possibility of technical changes across modes (and nodes). The GEMPACK code has a single nest, i.e. the first two equations are collapsed into a single equation (E_qtmf_{sd}). The GAMS code could eventually be extended to allow for substitution across modes as a function of relative prices. Equation (60) determines the aggregate price of international trade and transport services for each trade node. The expression relies on the price of each mode of transportation,

⁴⁴In the full GTAP Data Base, there are three modes of transportation—air, water and other transport.

which for lack of additional information, is assumed to be a global price and not specific to the trade node. Equation (61) determines the global demand for trade and transport services, $XTMG$, for each mode m .

$$XWGM_{r,i,d} = \zeta_{r,i,d}^{mg} XW_{r,i,d}^d \quad (58)$$

$$XMGM_{m,r,i,d} = \frac{\alpha_{m,r,i,d}^{mg}}{\lambda_{m,r,i,d}^{mg}} XWGM_{r,i,d} \quad (59)$$

$$PWMG_{r,i,d} = \sum_m \frac{\alpha_{m,r,i,d}^{mg}}{\lambda_{m,r,i,d}^{mg}} PTMG_m \quad (60)$$

$$XTMG_m = \sum_r \sum_i \sum_d XMGM_{m,r,i,d} \quad (61)$$

This subunit generates the variables $XWGM$, $XMGM$, $PWMG$ and $XTMG$, the latter three correspond to the GEMPACK variables `qtmfsd`, `ptrans` and `qtm`. There is no corresponding GEMPACK variable for $XWGM$.

Variable $XTMG$ represents global demand for trade and transport services by mode m . There is a global supplier that purchases these services across the regions of the world minimizing cost and using a CES production function. Equation (62) represents the demand for international trade and transport services for mode m sourced in region r . The substitution elasticity across suppliers is given by the elasticity σ^{mg} .⁴⁵ Note that the GAMS code generates a demand at the Armington level that will eventually be allocated across domestic and imported goods. Since import shares are zero in the base data, the Armington variable will be equal to the domestic component, corresponding to the variable `qst` in the GEMPACK code. Equation (63) determines the average global supply price for each mode m .⁴⁶

$$XA_{r,m,tmg} = \alpha_{r,m,tmg}^a XTMG_m \left(\frac{PTMG_m}{PA_{r,m,tmg}} \right)^{\sigma_{r,m}^{mg}} \quad (62)$$

⁴⁵The default value is a substitution elasticity of 1.

⁴⁶In the GEMPACK code, the component price is `pds` that corresponds to the GAMS supply price PS . The GAMS specification allows for two modifications. First, it assumes that the margin services are an Armington good. If instead the margin services were assumed to be explicitly a domestic good, the relevant component price would be PD . If, moreover, the GAMS code assumed perfect transformation of domestic output the relevant component price would be PS , the same as in the GEMPACK code. Note that the index `tmg` is a unitary subset of the set of Armington agents—representing the domestic margin service sector.

$$PTMG_m = \left[\sum_r \alpha_{r,m,tmg}^a PA_{r,m,tmg}^{(1-\sigma_{r,m}^{mg})} \right]^{1/(1-\sigma_{r,m}^{mg})} \quad (63)$$

This subunit determines the variables $XA_{r,m,tmg}$ and $PTMG_m$ corresponding to the GEMPACK variables qst and pt .

3.14 Bilateral trade prices

There are four bilateral trade prices corresponding to three price wedges. Producers in region r receive the price PE for commodity i delivered to region d . In the case of perfect transformation, the price PE is equal the aggregate supply price given by PS . Between the farm- or factory-gate, a bilateral export tax or subsidy (τ^e) is applied to the producer price and determines the export border price (or the free on board—FOB price), equation (64).⁴⁷ Equation (65) determines the import border price, PM^{cif} . Given the assumption of the Leontief demand for international trade and transport services, the import border price (or the cost, insurance and freight—CIF price) is equal to the FOB price augmented by the unit cost of the trade margin, equation (65). The final wedge represents the bilateral import tariffs (τ^m) that converts the CIF price of imports to the market bilateral price of imports, PM , equation (66).⁴⁸

$$PE_{r,i,d}^{fob} = PE_{r,i,d} (1 + \tau_{r,i,d}^e) \quad (64)$$

$$PM_{s,i,r}^{cif} = PE_{s,i,r}^{fob} + \zeta_{s,i,r}^{mg} PWMG_{s,i,r} \quad (65)$$

$$PM_{s,i,r} = PM_{s,i,r}^{cif} (1 + \tau_{s,i,r}^m) \quad (66)$$

This subunit generates PE^{fob} , PM^{cif} and PM corresponding to GEMPACK variables $pfob$, $pcif$ and $pmds$.

3.15 Market equilibrium

There are fundamentally two market equilibrium conditions for goods and services. The first guarantees equality of supply and demand for domestically produced goods sold on the domestic market. The second guarantees equality of supply and demand for each bilateral trade node. Equation (67) represents the equi-

⁴⁷In both the GEMPACK and GAMS versions, the code includes uniform shifters across trading partners for the bilateral export and import taxes. They are initialized at zero, but are intended for use in policy shock scenarios. See the variables `ETAX` and `MTAX` in the GAMS code.

⁴⁸This is not the final price of imports because a sales tax is applied to the aggregate import volume.

librium condition for the domestic market and essentially determines the equilibrium price PD . The supply side is determined from the CET domestic allocation specification. The demand side is determined by the top level Armington specification. Equation (68) reflects supply/demand equilibrium for each bilateral trade node, essentially determining the price PE . In the GAMS implementation, the latter equation is substituted out and the code only carries a variable XW without any superscripts.

$$XD_{r,i}^s = \sum_{aa} XD_{r,i,aa} \quad (67)$$

$$XW_{r,i,d}^s = XW_{r,i,d}^d \quad (68)$$

This subunit determines PD and PE . They have no equivalent in the GEMPACK code because they are linked to the CET specification of domestic production and are equal to the GAMS variable PS corresponding to the GEMPACK variable pds .

3.16 Factor markets

Factors are segmented into two groups. The first group allows for partial or perfect mobility of factors across sectors—governed by a CET specification. The second group of factors are sector specific. Note that in GEMPACK, there is a three way segmentation: partially mobile, or sluggish, with a CET specification (ENDWS), perfectly mobile (ENDWM), and sector specific (ENDWF).⁴⁹ The sector specific factor is treated differently from the CET with a zero transformation elasticity, because the sector specific resource is given an upward sloping supply curve. The set f of factors is therefore split into two subsets. One subset, fm , contains all mobile factors and will be governed by the CET specification with a transformation elasticity that can vary from 0 to ∞ . The second, fsm , contains factors that are treated as sector specific. Their sector-specific supply will be specified using an upward sloping supply curve (eventually with zero elasticity). The standard assumption is that labor and capital are perfectly mobile—i.e. with an elasticity of 'INF' in GAMS, associated with the subset ENDWM in GEMPACK. Land is partially mobile, with a transformation elasticity of 1.⁵⁰ And natural resources are sector specific with a default supply elasticity of 0.⁵¹

Equation (69) determines the aggregate supply of mobile factors, XFT . There is no equivalent in the GEMPACK code where the aggregate supply of all factors is exogenous. In this formulation, the supply curve is a function of the return to

⁴⁹The formulas in the GAMS version correspond to a positive transformation elasticity. In GEMPACK, the transformation elasticities are entered as a negative value (or zero).

⁵⁰In GEMPACK the transformation elasticity is entered as -1 and land is part of the ENDWS subset.

⁵¹There is no supply function in the GEMPACK version and the relevant sector supply is exogenous.

the aggregate factor, relative to an economy-wide price given by the variable $PABS$, which is a price index of domestic absorption and further described below. Setting the supply elasticity (η^{ft}) to zero would have the same impact as exogenizing total supply. Equation (70) determines the factor supply to each sector under one of three market specifications. The first two lines relate to mobile factors only. The first is the standard CET supply function for partially mobile factors (e.g. land). The second line holds for perfectly mobile factors (e.g. unskilled and skilled labor and capital in the standard GTAP model). With perfect mobility, the after-tax return of each factor is uniform across all sectors. The third line holds only for sector-specific factors such as natural resources. In this case supply is specified as an upward sloping supply curve with the possibility of a zero supply elasticity. Equation (71) determines the price of the aggregate factor bundle for mobile factors. Equation (72) represents the factor supply equilibrium condition, i.e. supply equals demand at the level of each sector (i.e. production activity). For perfectly mobile factors, the equilibrium condition will actually be represented by equation (71) that equates the sum of demand to total supply. The equilibrium condition is substituted out of the model and the superscripts are eliminated from the variable XF . The final two equations in this section, (73 and 74), link the equilibrium (or market) price of factors to the purchasers' (or agents') price of factors and to the after-tax return to households.

$$XFT_{r,fm} = A_{r,fm}^{ft} \left(\frac{PFT_{r,fm}}{PABS_r} \right)^{\eta_{r,fm}^{ft}} \quad (69)$$

$$\begin{cases} XF_{r,fm,a}^s = \gamma_{r,fm,a}^f XFT_{r,fm} \left(\frac{PF_{r,fm,a}^y}{PFT_{r,fm}} \right)^{\omega_{r,fm}^f} & \text{if } \omega_{r,fm}^f \neq \infty \\ PF_{r,fm,a}^y = PFT_{r,fm} & \text{if } \omega_{r,fm}^f = \infty \\ XF_{r,fm,a}^s = \gamma_{r,fm,a}^f \left(\frac{PF_{r,fm,a}^y}{PABS_r} \right)^{\eta_{r,fm}^{ff}} \end{cases} \quad (70)$$

$$\begin{cases} PFT_{r,fm} = \left[\sum_a \gamma_{r,fm,a}^f PF_{r,fm,a}^y (1+\omega_{r,fm}^f)^{(1+\omega_{r,fm}^f)} \right]^{1/(1+\omega_{r,fm}^f)} & \text{if } \omega_{r,fm}^f \neq \infty \\ XFT_{r,fm} = \sum_a XF_{r,fm,a}^s & \text{if } \omega_{r,fm}^f = \infty \end{cases} \quad (71)$$

$$XF_{r,f,a}^s = XF_{r,f,a}^d \quad (72)$$

$$PF_{r,f,a}^a = PF_{r,f,a} \left(1 + \tau_{r,f,a}^{ft} + \tau_{r,f,a}^{fs} \right) \quad (73)$$

$$PF_{r,f,a}^y = PF_{r,f,a} \left(1 - \kappa_{r,f,a}^f \right) \quad (74)$$

This subunit generates XFT , XF^s , PFT , PF , PF^a and PF^y . The corresponding GEMPACK variables are q_e , q_{es} , p_e , p_{eb} (market equilibrium price of factors supplied to activity a), p_{fe} (purchasers' price of factors in activity a) and p_{es} (after tax remuneration of factors used in activity a).

3.17 Investment behavior

Regional savings is determined by the top level utility function. All savings are collected by a 'global' saver that then allocates savings across the regions of the model thereby determining regional investment. There are several specifications for the behavior of the global saver. In the first specification, global savings are allocated across regions so as to equalize 'risk' adjusted 'expected' rates of return. In a second option, net new investment is allocated across regions using the same proportions as in the baseline. A third option, oft-used in CGE models, fixes the capital account. A fourth option fixes the ratio of net capital flows with respect to regional income. For accounting reasons, this can only be assumed for $n - 1$ regions and thus there is a residual region that is the lender/borrower of last resort. The last three options imply that deviations of risk adjusted rates of return can occur.

Equation (75) determines the level of the beginning of period capital stock. The GAMS version of the model carries two versions of regional capital stocks. The first is the 'normalized' level that equals aggregate capital remuneration in the base period. This is the notion of the capital stock that is allocated across sectors and whose price is equal to 1 in the base period. The 'non-normalized' level corresponds to the initial estimate of the value of the beginning of period capital stock. Typically, the ratio of the normalized level to the non-normalized level represents the gross rate of return to capital in the base period. The non-normalized level should also represent a low multiple of aggregate GDP (say between 2 and 3 for most regions), and its value should be compatible, in terms of units, with the volume of investment. The initial value for K^0 comes from the header array ∇KB and the parameter χ^k is calibrated using base period data. The GEMPACK code does not require both variables since it is expressed in terms of percent change—and both variables will have the same percentage change. Equation (76) calculates the end-of-period capital stock. It is equal to the depreciated level of the initial capital stock augmented by the in-period volume of investment (XI). Equation (76) is only valid with the non-normalized definition of the capital stock.

$$K_r^0 = \chi_r^k XFT_{r,cap} \quad (75)$$

$$K_r^1 = (1 - \delta_r) K_r^0 + XI_r \quad (76)$$

Equation (77) defines the after-tax return to aggregate capital. The numerator represents after-tax profits and the denominator is the beginning of period capital stock. Equation (78) defines the net return to capital after adjusting for the replacement cost of capital (similar to Tobin's q) and the rate of depreciation. Equation (79) defines the expected rate of return, R^e . It is equal to the contemporaneous net rate of return, R^c , adjusted by a factor that depends on future increases to the capital stock. As the capital stock increases, all else equal, one expects the return to decline. The level of adjustment depends on the elasticity ϵ^{RoR} .⁵²

$$R_r^a = \sum_a \left(1 - \kappa_{r,cap,a}^f\right) PF_{r,cap,a} XF_{r,cap,a} / K_r^0 \quad (77)$$

$$R_r^c = \frac{R_r^a}{PI_r} - \delta_r \quad (78)$$

$$R_r^e = R_r^c \left(\frac{K_r^1}{K_r^0}\right)^{-\epsilon^{RoR}} \quad (79)$$

This subunit defines K^0 , K^1 , R^a , R^c and R^e corresponding to the GEMPACK variables `kb`, `ksvces`, `rental`, `rorc` and `rore`.

There are four possible closure rules for investment.⁵³ Equation (80) reflects the specific closure for each of the four options, which is determined by the user-specified flag `RoRFlag`. With `RoRFlag` set to `capFlex`, closure is defined by the equality of the expected risk adjusted rates of return to the global rate of return, where π_r^f is the risk adjustment and calibrated with base year information.⁵⁴ With `RoRFlag` set to `capShrFix`, closure reflects that net investment growth across all regions is equal to the global growth in investment, or in other words, the global allocation of net investment reflects the initial allocation of net investment, where the parameter χ^I is calibrated to the initial volumes and is fixed. Note, that for consistency purposes, this form of the equation is defined over $n - 1$ regions. The third option, with `RoRFlag` set to `capFix`, fixes the capital account for each region as defined by the variable S^f . Again, the capital flows are fixed for only $n - 1$ of the regions and the capital account consistency equation will determine the capital

⁵²The elasticity ϵ^{RoR} corresponds to the `RORFLEX` parameter in the GEMPACK code. See [Hertel \(1997\)](#) for further elaboration.

⁵³In the standard GTAP model, there are two possible closures: capital flows respond to changes in expected rates of return (`capFlex`), and the fixed share allocation of global investment (`capShrFix`).

⁵⁴The GEMPACK version does not require the risk adjustment as in the percentage change formulation the risk adjustment factor drops out if it is exogenous.

flow for the residual region. The exogenous flows are given in volume terms. To preserve model homogeneity, the flows are valued at the global price of investment. The fourth option fixes net capital flows as a share of regional income. This can only hold for $n - 1$ regions and the net capital flow for the residual region will be driven by the capital account consistency equation.

$$\begin{cases} \pi_r^r R_r^e = R^g & \text{if RoRFlag} = \text{capFlex} \\ XI_r - \delta_r K_r^0 = \chi_r^I XI_{GBL} & \text{if RoRFlag} = \text{capShrFix} \\ S_r^f = PI_{GBL} \overline{S_r^f} & \text{if RoRFlag} = \text{capFix} \\ S_r^f = \chi_r^f RegY_r & \text{if RoRFlag} = \text{capSFix} \end{cases} \quad (80)$$

Equation (81) defines the global rate of return, R^g for all closures except when the equality of the expected rate of return mechanism is implemented (i.e. when $\text{RoRFlag} = \text{capFlex}$). It is defined as the weighted sum of the regional expected rates of return and the weights are provided by the regional shares of (net) investment in global (net) investment, equation (82). It is purely definitional and does not interact with the rest of the model. Equation (83) defines the ratio of net foreign capital flows with respect to regional income. The equation is dropped from the model if the capital account closure has this ratio as exogenous.

$$R^g = \sum_r \varphi_r^r R_r^e \quad \text{if } \text{RoRFlag} \neq \text{capFlex} \quad (81)$$

$$\varphi_r^r = \frac{PI_r (XI_r - \delta_r K_r^0)}{\sum_s PI_s (XI_s - \delta_s K_s^0)} \quad (82)$$

$$S_r^f = \chi_r^f RegY_r \quad \text{if } \text{RoRFlag} \neq \text{capSFix} \quad (83)$$

Equation (84) reflects the capital flow consistency requirement, i.e. the sum of net capital flows across all regions must be 0. With the expected rate of return closure, this equation in essence determines the global rate of return, R^g . With all other closures, this equation determines net foreign savings for the residual region. Equation (85) determines the level of gross nominal investment. It is equal to the sum of the value of depreciation (at replacement cost), regional savings, and net foreign capital flows.⁵⁵ Note, that due to Walras' Law, the equation is only specified for $n - 1$ regions.⁵⁶

⁵⁵In other words, net investment is equal to the sum of domestic and foreign savings.

⁵⁶The equation is included in the model definition for the residual region and includes a dummy variable named *Walras*. If the model solves correctly, this variable should have the value 0.

$$\sum_r S_r^f \equiv 0 \quad (84)$$

$$YI_r = \delta_r PI_r K_r^0 + Save_r + S_r^f \quad (85)$$

In summary, equation (80) holds for n regions for the so-called `capFlex` closure, otherwise holds for only $n - 1$ regions. Equation (84) holds for all closures. It 'solves' for R^s in the case of the flexible capital closure; otherwise it solves for the net capital flows of the residual region. Equation (85) solves for nominal regional investment for $n - 1$ regions, with one equation dropped due to Walras' Law. Equation (83) holds for all closures except for the fixed capital flow to regional income closure.⁵⁷

Equation (86) defines global (net) investment, XI_{GBL} , with equation (87) defining the average global price of purchasing investment goods, PI_{GBL} . The latter is used to value net foreign savings.

$$XI_{GBL} = \sum_r [XI_r - \delta_r K_r^0] \quad (86)$$

$$PI_{GBL} XI_{GBL} = \sum_r PI_r (XI_r - \delta_r K_r^0) \quad (87)$$

The savings deflator, used to evaluate the utility from savings, is defined in equation (88). In the absence of foreign capital flows, the regional price of savings would be equated to the regional price of investment. Even with foreign capital flows, the same logic suggests that the global price of savings should equate to the global price of investment. Nonetheless, foreign capital flows make the definition of the regional price of savings more complicated because net regional savings will be a combination of savings from other regions (that are not determined bilaterally). The definition below equates the regional price of savings to the regional price of investment, but applies an adjustment factor that ensures equality of the two price indices at the global level.⁵⁸

⁵⁷The model is equivalent to the GEMPACK version with the exception that the latter does not carry the variables YI nor S^f , which allows the GEMPACK version to drop equations (85) and (44). In effect, then, equation (80) determines XI , i.e. the *volume* of regional investment.

⁵⁸The GEMPACK version appeals to the same intuition but is implemented somewhat differently (private communication with Robert McDougall). The premise is that for well-behaved welfare results, the welfare effects of changes in investment goods prices and saving prices should balance. The form of the equation ensures that that holds approximately at the region level; the adjustment factor ensures that it holds exactly at the global level.

$$PSave_{r,t} = \chi^s \frac{PSave_{r,0}}{PI_{r,0}} PI_{r,t} \quad (88)$$

The adjustment factor, χ^s , is defined in the formula below. The formula depends on the regional investment and savings shares (relative to global levels), also provided below. Note that the weights rely on base year values.

$$\chi^s = \frac{\sum_r \zeta_r^I \frac{PI_{r,t}}{PI_{r,0}}}{\sum_r \zeta_r^S \frac{PSave_{r,t}}{PSave_{r,0}}}$$

$$\zeta_r^I = \frac{PI_{r,0} (XI_{r,0} - \delta_r K_{r,0}^0)}{\sum_s PI_{s,0} (XI_{s,0} - \delta_r K_{s,0}^0)}$$

$$\zeta_r^S = \frac{Save_{r,0}}{\sum_s Save_{s,0}}$$

This subunit generates S^f , R^s , φ^r , YI , XI_{GBL} , XI_{GBL} and $PSave$. These correspond to `rorg`, `qinv`, `globalcgds`, `pcgdswld` and `psave`.⁵⁹

3.18 Price indices and model numéraire

The latest version of the model has converted price indices to a Fisher price index, and similarly volume indices (such as GDP) to a Fisher volume index.⁶⁰ For a price index over set i , the Fisher price index is equal to the following:⁶¹

$$F^p = \sqrt{\frac{\sum_i P_{i,t} Q_{i,t-1}}{\sum_i P_{i,t-1} Q_{i,t-1}} \cdot \frac{\sum_i P_{i,t} Q_{i,t}}{\sum_i P_{i,t-1} Q_{i,t}}} = \sqrt{L^p \cdot P^p}$$

The Fisher price index represents the geometric mean of the Laspeyres (L^p) and Paasche price indices (P^p), with the former using lagged volume weights and the latter using current volume weights.

⁵⁹Though the variable S^f is not explicit in the GEMPACK code, the variable `TBAL` represents the trade balance in levels and given the definition of the balance of payments, it is equivalent to the negative of the capital account balance.

⁶⁰The same definition of price indices have been incorporated in the latest version of the GEMPACK code. After extensive testing this has eliminated any difference in model results up to 4-5 significant digits, with the exception of equivalent variation where the differences are small. We suspect this may be due to the definition of the price of savings, which differs across implementations. This has no impact on model results with the exception of defining the utility from savings.

⁶¹It is natural to think of the index 't' as time. However, in a comparative static framework it represents the pre- and post-shock values. The code reflects this fact in the sense that 't-1' is always set to 't0', i.e. the base period, in the case of comparative static simulations.

$$L^p = \frac{\sum_i P_{i,t} Q_{i,t-1}}{\sum_i P_{i,t-1} Q_{i,t-1}} \quad P^p = \frac{\sum_i P_{i,t} Q_{i,t}}{\sum_i P_{i,t-1} Q_{i,t}}$$

These indices represent percent change relative to the previous period and thus the price indices are chain weighted. The price index itself is thus defined as:

$$PI_t = PI_{t-1} F_t^p$$

where typically the price index will be set to 1 or 100 in some base period.

The Fisher volume index has a similar definition using different price weights:

$$F^q = \sqrt{\frac{\sum_i P_{i,t-1} Q_{i,t}}{\sum_i P_{i,t-1} Q_{i,t-1}} \cdot \frac{\sum_i P_{i,t} Q_{i,t}}{\sum_i P_{i,t} Q_{i,t-1}}} = \sqrt{L^q \cdot P^q}$$

The Fisher volume index represents the geometric mean of the Laspeyres (L^q) and Paasche volume indices (P^q), with the former using lagged price weights and the latter using current price weights.

$$L^q = \frac{\sum_i P_{i,t-1} Q_{i,t}}{\sum_i P_{i,t-1} Q_{i,t-1}} \quad P^q = \frac{\sum_i P_{i,t} Q_{i,t}}{\sum_i P_{i,t} Q_{i,t-1}}$$

It is easy to show that the aggregate nominal value is equal to the product of the price and volume indices:

$$P_t \cdot Q_t = \sum_i P_{i,t} \cdot Q_{i,t} = F_t^p \cdot F_t^q \cdot P_{t-1} \cdot Q_{t-1}$$

The equations below define four price indices—two of which are regional and two of which are global. Equation (90) defines a price index for regional absorption. It is based on the Fisher price index that uses the indicator *QABS* defined for prices for period *tp* and volumes for period *tq*. It represents the sum of Armington consumption using different years for price and volume weights and covers private, public and investment demand (as captured by the subset *fd* of *aa*.) Similarly, Equation (92) defines the Fisher price index of manufactured exports from high-income regions—intended to be closely related to the World Bank’s Manufactured Unit Value (MUV) index. It is based on the indicator *QMUV* which represents a volume of exports from designated regions over specified commodities. The first regional sum covers all regions defined in the subset *HIC*. The commodity index covers all commodities defined in the *MANU* index. The second regional sum is over all regions. The relevant export price is the border price, i.e. the FOB price.

$$QABS_{r,tp,tq} = \sum_i \sum_{fd} PA_{r,i,fd,tp} PA_{r,i,fd,tq} \quad (89)$$

$$PABS_{r,t} = PABS_{r,t-1} \sqrt{\frac{QABS_{r,t,t-1}}{QABS_{r,t-1,t-1}} \cdot \frac{QABS_{r,t,t}}{QABS_{r,t-1,t}}} \quad (90)$$

$$QMUV_{tp,tq} = \sum_{s \in HIC} \sum_{i \in MANU} \sum_d PE_{s,i,d,tp}^{fob} XW_{s,i,d,tq}^s \quad (91)$$

$$PMUV_t = PMUV_{t-1} \sqrt{\frac{QMUV_{t,t-1}}{QMUV_{t-1,t-1}} \cdot \frac{QMUV_{t,t}}{QMUV_{t-1,t}}} \quad (92)$$

Equation (94) defines a second regional price index and it is an index of factor prices (evaluated at basic prices). It is a Fisher index based on the indicator $QFACTR$, which represents total factor remuneration using different periods for prices and quantities. Equation (96) is a similar index, but is an index of global factor prices. Equation (97) defines the global model numéraire. The GEMPACK default is the global factor price index, but any global price index could replace $PFACT^w$, such as $PMUV$. The price itself is exogenous, but the model is nonetheless square as one of the investment equations is dropped.

$$QFACTR_{r,tp,tq} = \sum_f \sum_a PF_{r,f,a,tp} XF_{r,f,a,tq} \quad (93)$$

$$PFACT_{r,t} = PFACT_{r,t-1} \sqrt{\frac{QFACTR_{r,t,t-1}}{QFACTR_{r,t-1,t-1}} \cdot \frac{QFACTR_{r,t,t}}{QFACTR_{r,t-1,t}}} \quad (94)$$

$$QFACTW_{tp,tq} = \sum_r \sum_f \sum_a PF_{r,f,a,tp} XF_{r,f,a,tq} \quad (95)$$

$$PFACT_t^w = PFACT_{t-1}^w \sqrt{\frac{QFACTW_{t,t-1}}{QFACTW_{t-1,t-1}} \cdot \frac{QFACTW_{t,t}}{QFACTW_{t-1,t}}} \quad (96)$$

$$PNUM = PFACT^w \quad (97)$$

This subunit generates $PABS$, $PMUV$, $PFACT$ and $PFACT^w$. Equation (97) defining the numéraire in essence determines investment for the residual region that is left out of the model specification. The corresponding variables in the GEMPACK version are `pabs`, `pmuv`, `pfactor` and `pfactwld`.

3.19 Miscellaneous

This section describes some additional equations that can be used to change the closure of the model. Equations (98) and (99) define the sales tax rate on domestic

and imported expenditures across all Armington agents, respectively. Each has a commodity and agent-specific shifter (χ^{dtx} and χ^{mtx}), as well as a common shifter across all commodities and sources ($\chi_{r,aa}^{rtx}$). Equation (100) determines the share of the different tax streams relative to regional income.

$$\tau_{r,i,aa}^{dtx} = \tau_{r,i,aa,0}^{dtx} + \chi_{r,i,aa}^{dtx} + \chi_{r,aa}^{rtx} \quad (98)$$

$$\tau_{r,i,aa}^{mtx} = \tau_{r,i,aa,0}^{mtx} + \chi_{r,i,aa}^{mtx} + \chi_{r,aa}^{rtx} \quad (99)$$

$$YTAXShr_{r,gy} = \frac{YTAX_{r,gy}}{Y_r} \quad (100)$$

It is convenient to have macroeconomic indicators—particularly for targeting purposes, such as to target the investment to GDP ratio, or to target GDP itself. Equation (101) defines nominal GDP at market price. Note that the domestic supply of international transport margins is classified as an Armington agent and is part of the subset fd . Equation (102) defines a GDP indicator that is evaluated in prices of year tp with volumes of year tq , $QGDP$. When the two time indices are identical, the indicator represents nominal GDP for the specified year. Equation (103) defines real GDP at market price, $RGDPMP$. The square root term represents the Fisher volume index for GDP growth. Equation (104) defines the GDP at market price deflator, $PGDPMP$.

$$GDPMP_r = \sum_{fd} \sum_i PA_{r,i,fd} XA_{r,i,fd} + \sum_i \left[\sum_d PE_{r,i,d}^{fob} XW_{r,i,d} - \sum_s PE_{s,i,r}^{cif} XW_{s,i,r} \right] \quad (101)$$

$$\begin{aligned} QGDP_{r,tp,tq} &= \sum_{fd} \sum_i PA_{r,i,fd,tp} XA_{r,i,fd,tq} \\ &+ \sum_i \left[\sum_d PE_{r,i,d,tp}^{fob} XW_{r,i,d,tq} - \sum_s PE_{s,i,r,tp}^{cif} XW_{s,i,r,tq} \right] \end{aligned} \quad (102)$$

$$RGDPMP_{r,t} = RGDPMP_{r,t-1} \sqrt{\frac{GDPMP_{r,t}}{GDPMP_{r,t-1}} \cdot \frac{QGDP_{r,t-1,t}}{QGDP_{r,t,t-1}}} \quad (103)$$

$$PGDPMP_r = GDPMP_r / RGDPMP_r \quad (104)$$

The model includes equations that measure equivalent and compensating variation. The first measures the expenditure needed to achieve post-shock utility at base year prices. The latter measures the expenditure needed to achieve pre-shock utility at post-shock prices. Equation (105) defines EV , or the equivalent variation

linked to private demand. It is defined implicitly with the CDE utility function evaluated at base year prices and post-shock utility. Equation (106) implicitly defines compensating variation for private demand, CV , using the CDE utility function evaluated at post-shock prices and base utility. Due to the CES expenditure function, equivalent variation for government expenditure is simply the post-shock volume of government expenditures multiplied by the base government price deflator: $EVG = PG_0 \cdot YG/PG$. Similarly, equivalent variation for savings is given by $EVS = PSave_0 \cdot Save/PSave$. Aggregate equivalent variation is the sum of all three components. The values presented in Section 6 reflect aggregate equivalent variation. The levels are very close to those derived from the GEMPACK version. One possible source of difference comes from the expression for the price of savings, which does differ across the two implementations. The price of savings has no bearing on the model results with the single exception of measuring the utility from savings.

$$\sum_i \alpha_{r,i,h}^a PA_{r,i,h}^{b_{r,i}} \left(U_r^P \right)^{b_{r,i} e_{r,i}} \left(\frac{EV_r}{Pop_r} \right)^{-b_{r,i}} \equiv 1 \quad (105)$$

$$\sum_i \alpha_{r,i,h}^a PA_{r,i,h}^{b_{r,i}} \left(U_{r,0}^P \right)^{b_{r,i} e_{r,i}} \left(\frac{CV_r}{Pop_r} \right)^{-b_{r,i}} \equiv 1 \quad (106)$$

3.20 Technology

The model described above embeds five explicit variables that are used to drive technological changes in production. These are summarized in Table 2. In a typical application, these productivity variables are exogenous. However they can be modified to look at the impacts of productivity changes at either the regional or activity level—or even more detailed. The productivity factors, through specific shifters—described below—, can also be endogenized in order to achieve a particular target for another variable. For example, this type of procedure is often used to target a specific growth rate for GDP in the calibration of a baseline scenario.

Table 2. Technology variables

<i>Variable</i>	<i>GAMS variable</i>	<i>Description</i>
$A_{r,a}^{\lambda p}$	axp	Total productivity, i.e. across all inputs to production
$\lambda_{r,a}^{nd}$	lambdand	Productivity of aggregate intermediate demand bundle
$\lambda_{r,a}^{va}$	lambdava	Productivity of aggregate value added bundle, i.e. TFP
$\lambda_{r,f,a}^f$	lambdaf	Productivity of specific factors
$\lambda_{r,i,a}^{io}$	lambdaio	Productivity of specific intermediate inputs

Source: Author.

The model therefore incorporates additional equations that drive changes for the five productivity factors. These equations allow for productivity affects to be driven by specific channels, i.e. shifters,—region and activity specific, activity specific (but uniform across regions), or region specific (but uniform across activities).

Equation (107) defines the motion equation for the total productivity factor, A^{xp} .⁶² This productivity factor changes the production possibilities frontier uniformly across all inputs—both intermediate and factors. The δ parameters are initialized at 0 and are typically exogenous. The δ^{xa} has no regional index and is therefore a productivity shifter that is uniform across all regions. If one wanted to target a change in the global price of a specific activity a , the factor could be made endogenous and the global price exogenous.⁶³ Similarly, the shifter δ^{xr} is region specific and thus would operate uniformly across all activities in designated regions. The third and final shifter, δ^x , is both region and activity specific.

$$A_{r,a,t}^{xp} = A_{r,a,t-1}^{xp} [1 + \delta_{a,t}^{xa} + \delta_{r,t}^{xr} + \delta_{r,a,t}^x] \quad (107)$$

Equations (108) and (109) apply to the top level production nests and allow for bundle specific shifts in the productivity factor, i.e. for the aggregate intermediate demand bundle, ND , and the aggregate value added bundle, VA . Similar to equation (107), these equations have three individual shifters that can apply across all activities (globally), across all regions (uniformly across activities), or be region and activity specific. Note that λ^{va} is typically associated with total factor productivity (TFP), as it applies uniformly across all factors of production.

$$\lambda_{r,a,t}^{nd} = \lambda_{r,a,t-1}^{nd} [1 + \delta_{a,t}^{na} + \delta_{r,t}^{nr} + \delta_{r,a,t}^n] \quad (108)$$

$$\lambda_{r,a,t}^{va} = \lambda_{r,a,t-1}^{va} [1 + \delta_{a,t}^{va} + \delta_{r,t}^{vr} + \delta_{r,a,t}^v] \quad (109)$$

The final set of equations apply to individual inputs. Equation (110) applies to individual intermediate demand inputs. For example, improvements in energy efficiency can be implemented for the energy inputs and differentiated across industries. There are four shifters—uniform for input i across all activities and regions, uniform for activity a across all intermediate inputs and regions, uniform across all regions for each input and activity and specific to each input in each activity in each region. Equation (111) applies to the individual factors of production. There are five possible shifters that mostly have the same interpretation as above. The fourth deserves some additional explanation—it applies uniformly across all factors of production, irrespective of activity. However, the indicator flag, κ^f , can be used to limit its impact across factors and its individual cells take (0,1) values. If all cells take the value 1, the shifter is applied to all factors of production. The purpose of this shifter is to allow for factor-specific productivity increases, for example

⁶²The motion equation assumes step sizes of 1 (i.e. 1 year). Models with variable step sizes would need to replace this equation with $\lambda_t = \lambda_{t-n} (1 + \delta)^n$, where n is the step size.

⁶³This would require some additions to the model as there is no equation in the standard version that defines a global price for activity a .

labor-augmenting. For the latter, κ^f should take the value 0 for all factors with the exception of labor.⁶⁴

$$\lambda_{r,i,a,t}^{io} = \lambda_{r,i,a,t-1}^{nd} \left[1 + \delta_{i,t}^{ii} + \delta_{a,t}^{ia} + \delta_{r,t}^{ir} + \delta_{r,i,a,t}^i \right] \quad (110)$$

$$\lambda_{r,f,a,t}^f = \lambda_{r,f,a,t-1}^f \left[1 + \delta_{f,t}^{ff} + \delta_{a,t}^{fa} + \delta_{r,f,t}^{fr} + \kappa_{r,f}^f \delta_{r,t}^{fr} + \delta_{r,f,a,t}^f \right] \quad (111)$$

This subunit generates A^{xp} , λ^{va} , λ^{io} , λ^f and λ^{nd} . The corresponding variables in the GEMPACK version are respectively `aoworld`, `ava`, `afa`, `af` and `aint`.

3.21 Summary of price relations

This section summarizes the multiple relations across all prices. For the goods markets there are two sets of equilibrium prices from which all other prices can be derived. These are the markets for goods produced domestically and sold on the domestic market, PD , and the bilateral export prices, PE . Figure 5 illustrates the downstream linkages between the price of domestic output and the subsequent market prices. In the presence of constant-returns-to-scale technology and perfect competition, the unit cost of production, PX , also represents the pre-tax producer price. Domestic output, XP , is converted to domestic supply, XS , using the 'make' matrix, with a corresponding transformation of the output price PX to the price of aggregate domestic supply PS . The basic price of the 'make' commodities is P that is converted to the purchasers' price, PP , when adjusted for the output tax, τ^p . Domestic supply is sold to both domestic and export markets. The price on the former is PD . The producer receives price PE from each destination region—though due to the nested nature of the allocation of aggregate output, there is an intermediate aggregate price of exports represented by PET . Between the farm- or factory-gate, there is an additional tax or subsidy on exports, τ^e , that generates the border price of exports, PE^{fob} . From the source border to the destination border, transportation and trade margins are tacked on, as represented by the margin wedge, ζ^{mg} , leading to the border price of imports, PM^{cif} . The bilateral market of imports, PM , is equal to the border price adjusted by import taxes.

The summary of prices from the demand side will pick up from the border price of imports, PM^{cif} , and the market price of domestic production sold on the domestic market, PD , figure 6. The bilateral market price of imports, PM , is equal to the border price adjusted by import taxes. Imports are aggregated cross source regions to form an aggregate import composite good with a price given by PMT . Agent-specific sales tax are then applied to domestic goods and the aggregate import good to form their end-user prices, respectively PD^p and PM^p . These last prices are then

⁶⁴The GEMPACK version of the standard model does not have this indicator flag, and instead, if this shifter is used (`afereg` in the TABLO code), it is applied to all factors.

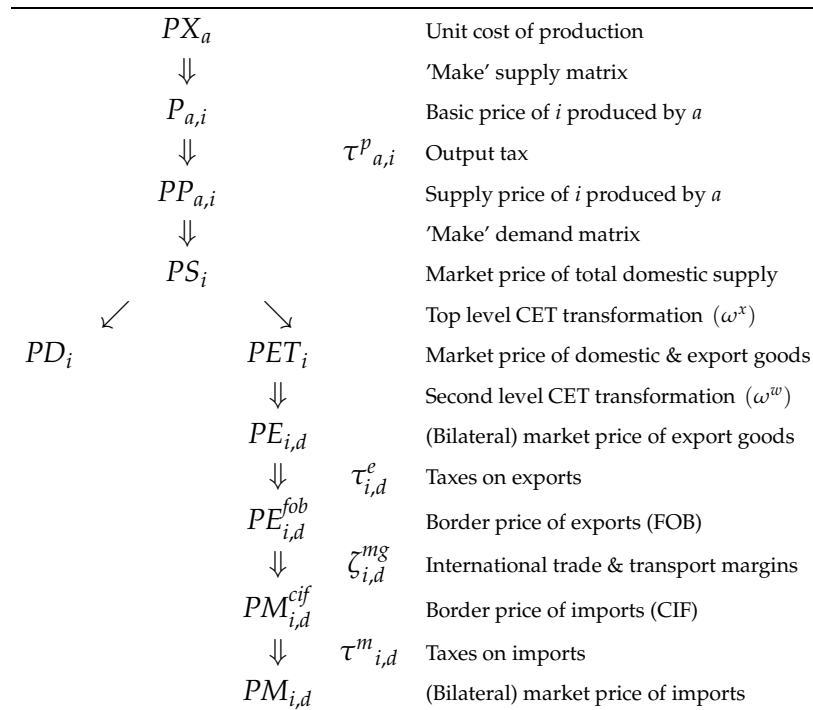


Figure 5. Summary of prices from the supply side

used to form domestic absorption, XA , with a price of PA that represents the aggregate, tax-inclusive price of domestic and imported goods.

The remaining key price wedge in the model regards factor prices. The equilibrium (or market) price for factors is given by PF . The purchasers' price of factors (or the cost of factors to producers), is given by PF^a , which is the market price adjusted by factor taxes and subsidies as represented by τ^{ft} and τ^{fs} , respectively. The after-income tax price of factors is given by PF^y .

3.22 GEMPACK Implementation

This section provides a brief description of some of the key differences between the GAMS implementation of the standard GTAP model and the GEMPACK version.

Levels versus percent change

In the GAMS version, all variables are handled in levels. The key advantage is the transparency between economic theory and its implementation, i.e. the one-to-one correspondence between the expression of the economic theory in its algebraic formulation and its implementation in software. GEMPACK requires an additional step that converts the algebraic equations in levels into their log-differentiated expression. For many behavioral equations, the log-differentiation is an advantage

	$PM_{s,i}^{cif}$		Border price of imports (CIF)
	↓	$\tau_{s,i}^m$	Taxes on imports
	$PM_{s,i}$		(Bilateral) market price of imports
	↓		Second level Armington CES (σ^w)
PD_i	PMT_i		Market price of domestic & (aggregate) import goods
↓	↓	$\tau_i^{dtx}, \tau_i^{mtx}, \chi_i^{tx}$	Sales tax on domestic & imported goods
PD_i^p	PM_i^p		End-user price of domestic & (aggregate) import goods
↘	↙		Top level CES aggregation of domestic & import goods (σ^m)
	PA_i		Armington price of goods (by agent)

Figure 6. Summary of prices from the demand side

as in most cases, analysts are interested in percent changes to key variables and the log-differentiated form allows for ready interpretation of the end results from a simulation. Another key advantage of the log-differentiated form of a model is that it bypasses the calibration step of a model expressed in levels. As shown below, the share parameters of the CES drop out of the CES equations. On the other hand, most accounting identities in the log-differentiated form require calculating (and updating) component shares, which is not needed in the levels version of a model.⁶⁵

In its most general form, the CES formulas are expressed by the following two expressions, where V is the aggregate volume with a price PV , and the components of the CES are X_i with a price of P_i . The (dual) share parameters are given by α_i . A represents an aggregate shifter in the preference function and λ_i are component specific shifters.⁶⁶ The first expression represents the demand function for component i . It is a share of the total volume V , where the share reflects a degree of price sensitivity that is governed by the substitution elasticity, σ . A value of zero represents a Leontief technology if used in a production function. The second expression represents the dual price expression of the CES. It can be shown that it is equal to the aggregate price of V , i.e. the price that represents the zero profit condition.⁶⁷

$$X_i = \alpha_i (A\lambda_i)^{\sigma-1} \left(\frac{PV}{P_i} \right)^\sigma V$$

⁶⁵A much more in-depth comparison of the two approaches, including a third approach (MPSGE), is available in [Horridge et al. \(2013\)](#).

⁶⁶For example productivity or preference shifters.

⁶⁷The zero profit condition is given by $PV.V = \sum_i P_i X_i$. The two expressions can be used interchangeably.

$$PV = \frac{1}{A} \left[\sum_i \alpha_i \left(\frac{P_i}{\lambda_i} \right)^{(1-\sigma)} \right]^{1/(1-\sigma)}$$

The log-differentiation of these two expressions leads to the two formulas below. The economic interpretation is relatively straightforward (ignoring the impacts of technical changes). Demand for component X_i rises in proportion with the rise in aggregate volume V , the so-called scale effect, that is adjusted by the price effect given by $\sigma(\dot{P} - \dot{P}_i)$. If the component price, P_i is rising relative to the average price, P , the price effect will reduce demand for component X_i , and the price effect is stronger the higher the substitution elasticity. Again ignoring the productivity factors, the aggregate price expression equates the percent change in the aggregate price to the share-weighted percent change in the component prices. If component i has a 50% share and its price rises by 10%, the aggregate price will rise by (around) 5%.

$$\dot{X}_i = \dot{V} + \sigma(\dot{P} - \dot{P}_i) + (\sigma - 1)(\dot{A} + \dot{\lambda}_i)$$

$$\dot{P} = -\dot{A} + \sum_j s_j (\dot{P}_j - \dot{\lambda}_j)$$

Many of the equations in the standard GTAP model are some form of the CES and CET functions and thus the two expressions above can be used to easily translate from the levels form of the equations to the percent difference forms. For example, the demand for factors equation in the GEMPACK version of the model is given by:

```

1  Equation E_qfe
2  # demands for endowment commodities #
3  (all,e,ENDW) (all,a,ACTS) (all,r,REG)
4  qfe(e,a,r)
5  = -afe(e,a,r) + qva(a,r)
6  - ESUBVA(a,r) * [pfe(e,a,r) - afe(e,a,r) - pva(a,r)];

```

which corresponds to equation (4). The variable `qfe` corresponds to X_i above, `qva` corresponds to V , `pva` corresponds to PV , `pfe` corresponds to P and `afe` corresponds to λ .⁶⁸ The substitution elasticity is given by `ESUBVA`. Similarly, the price of the aggregate value added bundle is given by the following GEMPACK equation and corresponds to equation (5):

```

1  Equation E_pva
2  # effective price of primary factor composite in each sector/region #
3  (all,a,ACTS) (all,r,REG)

```

⁶⁸The demand for factors equation does not have an aggregate shifter corresponding to A .

$$4 \quad pva(a, r) = \text{sum}\{e, \text{ENDW}, \text{VASHR}(e, a, r) * [pfe(e, a, r) - afe(e, a, r)]\};$$

The coefficient *VASHR* represents the revenue share of factor *e* in total factor remuneration in sector *a* in region *r*. It is updated at each iteration.

Taxes in GEMPACK

GEMPACK taxes are handled using so-called power variables. Thus, the output, or market price equation in the GEMPACK formulation would be:

$$PP = T.PB$$

where *T* is defined as:

$$T = (1 + \tau^p)$$

This has the advantage that the tax variable can be handled as a regular variable since it is centered around 1 (whereas a tax rate is centered around 0 and can cause problems when calculating percent changes). In percentage change terms, the price equation takes the following simple form:

$$pp = pb + t$$

Note that the power of a tax is greater than 1 for a positive tax and less than one for a subsidy.

In addition, the tax revenue equations are described in ordinary change format and relative to regional income. If *TY* represents a tax revenue stream, the relevant equation in the GEMPACK version will specify it as *TY/Y*, where *Y* is regional income, and as an ordinary change. This and other GEMPACK features are described more fully in the mathematical appendix found in [Corong et al. \(2017\)](#).

4. Dynamics

This chapter introduces some relatively simple additions to the standard model to make it a recursive dynamic model. There are two essential elements to dynamics: updating of the endowments—typically fixed within a time period—and updating of productivity parameter(s).⁶⁹

For our purposes, we have added two equations to the standard model definition. The first defines the growth of real GDP in per capita terms, g^y . In most simulations, Equation (112) can be used to determine the per capita growth rate of GDP as GDP is typically endogenous. In a baseline, the equation can be used to

⁶⁹The GEMPACK model could be modified in a similar fashion to convert it to a recursive dynamic framework. One possible advantage of the GAMS version is the time-subscripting of all variables and equations.

target real GDP growth and some instrument, i.e. variable, is used to achieve the targeted GDP growth rate. Given that there is a single target (per region), the instrument must also be economy-wide, for example an economy-wide productivity parameter such as TFP.

We define an additional equation that provides one specification for determining productivity growth combined with targeting GDP. Equation (113) determines the productivity factor δ^f defined earlier that is one of the factors determining factor productivity growth, λ^f , equation (111). First, it is defined only over labor endowments, indexed by l . Second, it is potentially differentiated across labor skills and activities. The variable γ^l is an economy-wide labor productivity factor that can be the instrument used to target GDP growth in a baseline scenario. The parameters π^a and π^m allow for activity and labor specific differentiation of labor productivity, relative to the economy-wide variable γ^l . These parameters can be gleaned from the existing literature on labor productivity differentials across sectors (if not skills). For example, one could set π^m to 1, and π^a to 0.01 in agriculture and 0.02 in manufacturing. The variable γ^l then reflects labor productivity in the services sectors. With these assumptions, labor productivity will be 1 percentage point higher in agriculture than in services and 2 percentage points higher in manufacturing than in services.

$$RGDPMP_{r,t} = (1 + g_{r,t}^y) RGDPMP_{r,t-1} Pop_{r,t} / Pop_{r,t-1} \quad (112)$$

$$\delta_{r,l,a,t}^f = \pi_{r,l,a,t}^a + \pi_{r,l,a,t}^m \gamma_{r,t}^l \quad (113)$$

A simple dynamic framework can be shaped by the following elements:

- 1) Update population between periods. In most cases population will continue to be exogenous within a period. The population update can be defined across age cohorts, but at a minimum will need to be defined for total population. Future projections of population at the global level are available from the United Nations Population Division that issues periodic revisions⁷⁰ and also from the demographic group at the International Institute for Applied Systems Analysis (IIASA).⁷¹ IIASA has been involved in the development of the relatively new set of long-term scenarios for climate change analysis commonly referred to as the Shared Socio-Economic Pathways (SSPs). See the 2017 special section of issue 42 of *Global Environmental Change* for a suite of articles on the SSPs (van Vuuren et al. (2017)).

⁷⁰The latest revision, 2017 *Revision of World Population Prospects* (<https://esa.un.org/unpd/wpp/>), was released in June 2017.

⁷¹<http://www.iiasa.ac.at/web/home/research/researchPrograms/WorldPopulation/Introduction.html>.

- 2) Update labor endowments. Growth in the labor supply is typically associated with the growth of the working age population, often the age cohort between 15-64 years of age. But other more detailed assumptions can be made including age-specific labor force participation rates, skill-specific growth rates (based on projections of educational attainment levels) and assumptions regarding migration (internal, for example rural to urban, as well as international).
- 3) Capital endowments are usually determined by the standard capital accumulation motion equation, see for example equation (76). In fact, the beginning of period capital stock can be set to the previous period's end of period capital stock, i.e. K^1 . Capital accumulation will be influenced by savings decisions, and regional savings rates may need to adjust to provide a plausible path for rates of return.
- 4) Baseline targets for GDP growth need to be specified. These could come from international agencies such as the International Monetary Fund (IMF), the Organisation for Economic Co-operation and Development (OECD) and/or the World Bank—though often these are short- and medium term projections. Alternatively one could use the set of international scenarios such as the SSPs referred to above. Three separate institutions have developed economic scenarios for the 5 SSPs—all harmonized to the same set of population scenarios developed at IIASA. The three institutions are IIASA, OECD and the Potsdam Institute for Climate Impact Research (PIK). PIK's GDP projections are only available for a proscribed 32-region aggregation of the world's economies. Given the GDP targets, the baseline is used to calibrate the relevant economy-wide productivity parameter as described above, or using some other specification.
- 5) It may be desirable to alter other productivity and preference parameters—for example energy efficiency, transport margins, preferences for imports, etc.

5. Accounting framework

This section describes the accounting framework used by the GAMS version of the model.⁷² Two different frameworks will be described. The first describes the analytical SAM underlying the model, i.e. how the SAM can be constructed using the variables of the model. The second describes the links between the model variables and the initial database as provided by GTAP. The depicted SAM is not a standard SAM and does not represent the full functionality of the model nor the underlying database. For example, demand is specified at the Armington level and the sales tax are collapsed to a single row dimension.

⁷²The underlying accounting framework of the GEMPACK version is similar, albeit using different naming conventions (Corong et al. (2017)).

5.1 The analytical SAM

The analytical SAM is the accounting framework that is derived from the variables of the model. In the absence of a shock, the analytical SAM should reproduce the input SAM. Table 3 re-produces the analytical SAM. Below are a few explanatory notes:

- 1) The regional index (r) is dropped for most of the expressions except when needed, for example in the bilateral trade flows.
- 2) The SAM departs from standard accounting principles in a number of ways. First, the law of one price does not hold across all rows. For example, the activities' rows contains the value of Armington consumption at pre-tax prices. The market prices of domestic goods, PD , and aggregate imported goods, PMT , are uniform across all agents, but not the Armington price.
- 3) For the sake of exposition, the final demand columns have been collapsed into a single column labeled FDM that covers the final demand index fd . Taken literally, this implies that the analytical SAM is not square as the revenue rows for the final demand agents are fully specified. However, all of the entries of the final demand expenditure accounts have the same generic formula.
- 4) The SAM reflects the differentiation between activities and commodities. The intersection of the activity rows with the commodity columns represents the transformation of domestic production into commodities. In the standard version of the model, the matrix is diagonal and there is a one-to-one mapping between activities and commodities.
- 5) The regional household is explicit in this version of the SAM. It could be dropped in which case there would need to be balancing items across the household, government and investment accounts.
- 6) The trade rows under the commodity columns represents the bilateral imports for region r . These are priced at border, i.e. CIF, prices. The commodity rows under the trade columns represent bilateral exports from region r and are valued at border, i.e. FOB, prices. At the global level, the difference between the two matrices represents the aggregate value of the trade and transport margins.
- 7) The BOP row under the trade columns represents aggregate imports from each region (at border prices). Similarly, the trade rows under the BOP column represents the aggregate exports of region r towards each region. The BOP accounts reflect the full balance of payments. For each region, the sum of exports (including the export of trade and transport services) less the sum of imports equals the net flow of foreign savings. At the global level, the foreign savings cell should be zero.

Table 3. Analytical SAM

	<i>ACT(a)</i>	<i>COMM(i)</i>	<i>FCT(f)</i>
Activities(a)		$P_{a,i}X_{a,i}$	
Commodities(i)	$PD_iXD_{i,a} + PMT_iXM_{i,a}$		
Factors(f)	$PF_{f,a}XF_{f,a}$		
Indirect tax	$\sum_i [\tau_{i,a}^d PD_iXD_{i,a} + \tau_{i,a}^m PMT_iXM_{i,a}]$		
Production tax	$\tau_a^p PX_aXP_a$		
Factor tax	$\sum_f \tau_{f,a}^{ft} PF_{f,a}XF_{f,a}$		
Factor subsidy	$\sum_f \tau_{f,a}^{fs} PF_{f,a}XF_{f,a}$		
Import tax		$\sum_s \tau_{s,i,r}^m PM_{s,i,r}^{cif} XW_{s,i,r}$	
Export tax		$\sum_d \tau_{r,i,d}^e PE_{r,i,d} XW_{r,i,d}$	
Direct tax			$\kappa_f^f \sum_a PF_{f,a}XF_{f,a}$
Regional household			$(1 - \kappa_f^f) \sum_a PF_{f,a}XF_{f,a} - \delta.PI.K^0$
Private			
Government			
Investment			$\delta.PI.K^0$
Trade margins			
Trade(s)		$PM_{s,i,r}^{cif} XW_{s,i,r}$	
Balance of payments			

Source: Author.

Table 3 Analytical SAM, ctd.

	<i>ITX</i>	<i>PTX</i>	<i>FTX</i>	<i>FTS</i>	<i>MTX</i>	<i>ETX</i>	<i>DTX</i>	<i>RHH</i>
<i>Activities(a)</i>								
<i>Commodities(i)</i>								
<i>Factors(f)</i>								
<i>Indirect tax</i>								
<i>Production tax</i>								
<i>Factor tax</i>								
<i>Factor subsidy</i>								
<i>Import tax</i>								
<i>Export tax</i>								
<i>Direct tax</i>								
<i>Regional household</i>	$YTAX_{at}$	$YTAX_{pt}$	$YTAX_{ft}$	$YTAX_{fs}$	$YTAX_{mt}$	$YTAX_{et}$	$YTAX_{dt}$	
<i>Private</i>								YC
<i>Government</i>								YG
<i>Investment</i>								Save
<i>Trade margins</i>								
<i>Trade(s)</i>								
<i>Balance of payments</i>								

Source: Author.

Table 3 Analytical SAM, ctd.

	FDM(<i>fd</i>)	TRD(<i>d</i>)	BOP
Activities(<i>a</i>)			
Commodities(<i>i</i>)	$PD_iXD_{i,fd} + PMT_iXM_{i,fd}$	$PE_{r,i,d}^{fob}XW_{r,i,d}$	
Factors(<i>f</i>)			
Indirect tax	$\sum_i \left[\tau_{i,fd}^d PD_iXD_{i,fd} + \tau_{i,fd}^m PMT_iXM_{i,fd} \right]$		
Production tax			
Factor tax			
Factor subsidy			
Import tax			
Export tax			
Direct tax			
Regional household			
Private			
Government			
Investment			S^f
Trade margins			$\sum_m PA_{m,tmg}XA_{m,tmg}$
Trade(<i>s</i>)			$\sum_i PE_{r,i,s}^{fob}XW_{r,j,s}$
Balance of payments		$\sum_i PM_{d,i,r}^{cif}XW_{d,j,r}$	

Source: Author.

5.2 Correspondence to GTAP Data Base

Table 4 provides the correspondence between the variables in the GAMS model and the standard GTAP Data Base that is used as the starting basis for the GTAP model.

Table 4. Correspondence between GAMS model variables and GTAP Data Base

GTAP	GAMS	Description
VIFB	$PMT_{r,i}XM_{r,i,a}$	Firms' import purchases at basic prices
VDFB	$PD_{r,i}XD_{r,i,a}$	Firms' domestic purchases at basic prices
VIFP	$PM_{r,i,a}^a XM_{r,i,a}$	Firms' import purchases at purchasers' prices
VDFP	$PD_{r,i,a}^a XD_{r,i,a}$	Firms' domestic purchases at purchasers' prices
VIPB	$PMT_{r,i}XM_{r,i,h}$	Private import purchases at basic prices
VDPB	$PD_{r,i}XD_{r,i,h}$	Private domestic purchases at basic prices
VIPP	$PM_{r,i,h}^a XM_{r,i,h}$	Private import purchases at purchasers' prices
VDPP	$PD_{r,i,h}^a XD_{r,i,h}$	Private domestic purchases at purchasers' prices)
VIGB	$PMT_{r,i}XM_{r,i,gov}$	Government import purchases at basic prices
VDGB	$PD_{r,i}XD_{r,i,gov}$	Government domestic purchases at basic prices
VIGP	$PM_{r,i,gov}^a XM_{r,i,gov}$	Government import purchases at purchasers' prices
VDGP	$PD_{r,i,gov}^a XD_{r,i,gov}$	Government domestic purchases at purchasers' prices)
VIIB	$PMT_{r,i}XM_{r,i,inv}$	Investment import purchases at basic prices
VDIB	$PD_{r,i}XD_{r,i,inv}$	Investment domestic purchases at basic prices
VIIP	$PM_{r,i,inv}^a XM_{r,i,inv}$	Investment import purchases at purchasers' prices
VDIP	$PD_{r,i,inv}^a XD_{r,i,inv}$	Investment domestic purchases at purchasers' prices
EVFB	$PF_{r,f,a}XF_{r,f,a}$	Firms' factor purchases at basic prices
EVFP	$PE_{r,f,a}^a XF_{r,f,a}$	Firms' factor purchases at purchasers' prices
FBEP	$-[\tau_{r,f,a}^s PF_{r,f,a}XF_{r,f,a}]$	Value of factor demand subsidies
FTRV	$\tau_{r,f,a}^t PF_{r,f,a}XF_{r,f,a}$	Value of factor demand taxes
VOS	$PP_{r,a}XP_{r,a}$	Value of production at producer cost
MAKB	$P_{r,a,i}X_{r,a,i}$	Value of make matrix at basic prices
MAKS	$PP_{r,a,i}X_{r,a,i}$	Value of make matrix at suppliers' prices
EVOS	$(1 - \kappa_{r,f}^f) \sum_a PF_{r,f,a}XF_{r,f,a}$	After tax factor remuneration
VKB	K_r^0	Beginning of period capital stock
VDEP	$\delta_r PI_r K_r^0$	Value of depreciation
SAVE	$Save_r$	Domestic savings
VIMB	$PM_{s,i,r}XW_{s,i,r}$	Value of bilateral imports tariff inclusive
VCIF	$PM_{s,i,r}^{cif}XW_{s,i,r}$	Value of bilateral imports at border prices
VFOB	$PE_{r,i,d}^{fob}XW_{r,i,d}$	Value of bilateral exports at border prices
VXSB	$PE_{r,i,d}XW_{r,i,d}$	Value of bilateral exports at producer prices
VST	$PD_{r,m}XD_{r,m,img}$	Domestic exports of trade & transport services
VTWR	$PTMG_m XMGM_{m,r,i,d}$	Value of margins from r to d for good i using mode m
POP	Pop_r	Population—millions

Source: Author.

6. Model extensions

This section illustrates numerical results from running simulations with the GAMS version of the model, including the impacts of using alternative specifica-

tions that extend the standard GTAP model.⁷³ We will simulate the global removal of tariffs using the standard model specification and a number of alternative specifications. The GAMS version of the GTAP model has been tested and compared with the GEMPACK version and the model results are identical to within standard numerical precision (4-5 significant digits)—albeit with a limited aggregation, i.e. the 10x10 database⁷⁴ and with the standard model specification. The GAMS version includes a few extensions—most of which require additional inputs such as supply and transformation elasticities. The extensions, enumerated at the beginning, are repeated here:⁷⁵

- 1) A double-nested CET transformation structure for the allocation of domestic output by destination region. The GEMPACK version assumes perfect transformation, i.e. the law-of-one-price holds for all destination markets.
- 2) Supply curves for economy-wide and sector-specific factors. The GEMPACK version assumes factor supplies are exogenous, i.e. implicitly the supply elasticities are zero.⁷⁶
- 3) The GAMS version includes two additional capital account closure specifications, on top of GEMPACK's two specifications.

6.0.1 Limiting output response

Many models of international trade include a transformation function to allocate domestic output between domestic and export markets. This is one of the key features of the World Bank's 1-2-3 model.⁷⁷ One of the first implementations of the CET in a multi-regional CGE model was provided by the OECD's WALRAS project back in 1990.⁷⁸ This specification has also been implemented in the World Bank's LINKAGE and ENVISAGE models. Appendix B provides some key analytical insights into how implementing the CET for output allocation impacts trade and welfare.

In the GAMS version described herein, the CET is implemented analogously to the nested CES Armington structure. A first level nest allocates domestic output,

⁷³The GAMS version of the standard GTAP model (and the sample 10x10 database) are available in the supplementary files published with this article. Users, with the appropriate license for the GTAP Data Base, can prepare their own aggregation of the database using the GTAPAgg2 program available with the purchase of the database. Note that a GAMS license is also required to run the model.

⁷⁴Annex D provides the concordances for regions and sectors used for this exercise.

⁷⁵Some other code changes compared with the GEMPACK version do not change the fundamental model specification—for example the pooling of all agents of the economy into a single Armington set.

⁷⁶Factor supply curves and the additional capital account closures have been added to the recursive dynamic version of the GEMPACK model. These extensions will be made public in the near future.

⁷⁷Devarajan et al. (1990) and Devarajan et al. (1997).

⁷⁸van der Mensbrughe et al. (1990).

XP , between the domestic market, XD , and an aggregate export bundle, XET , see Figure 4. A second CET nest allocates aggregate exports across destination markets, thereby determining the supply of bilateral exports, XW^s .

Purely for expositional purposes, we are calibrating the tariff removal simulation with a uniform value of 2 for the first level CET elasticity, and a uniform value of 4 for the second level CET elasticity.⁷⁹

6.0.2 Capital account

The standard GTAP model has two capital account closure rules: 1) savings are allocated across regions in response to changes in the 'expected' regional rates of return to capital; and 2) the allocation shares of global investment across regions are fixed. We add to these two closures an additional two: 1) the capital account is fixed in real terms (relative to the global price of investment); and 2) the ratio of the capital account to regional income is fixed. Both of these assumptions are assumed to hold for $R - 1$ regions and there is a residual region that is the lender or borrower of last resort. We only test the first alternative herein.

6.0.3 Elastic supply curves for land and natural resources

The model allows for elastic supply curves for all endowments. Herein we will test separately adding a land supply curve and a natural resource supply curve. In principle, these two operate differently in the model. The land supply curve affects the aggregate land endowment, which is in turn allocated across land-using activities with a CET specification, i.e. it is a sluggish endowment. Each natural resource, which is sector specific, has its own supply curve and thus requires a separate elasticity. In our specific aggregation we have only one sector that uses a single natural resource.

In summary we will be performing the same shock, global free trade, under 5 model specifications, summarized in Table 5. The default specification assumes perfect transformation of domestic output across markets (i.e. both transformation elasticities default to a value of 'INF'), a flexible capital account that reacts to changes in cross-regional rates of return to capital (option 'capFlex'), and both (aggregate) land and natural resource supply elasticities are set at their default value of 0.

The bottom line results are summarized in Tables 6 and 7, which reflect the change in equivalent variation (relative to the no-policy shock base equilibrium).⁸⁰ Global real income increases by \$113 billion under the default model specification,

⁷⁹The relevant elasticities are named ω_{gax} and ω_{gaw} , respectively. They are initialized to INF in the 'getData.gms' file. Users can override the elasticities in the simulation file, in a statement before the model is calibrated, i.e. before including the file 'cal.gms'. See file 'GFTCET.gms'.

⁸⁰Equivalent variation is measured as the sum of the expenditure function for private and public expenditures and savings—defined as the expenditure necessary to achieve contemporaneous utility at base prices.

Table 5. Summary of simulations

Acronym	Description
GFT	Global free trade with standard model specification
GFTCET	Global free trade with domestic output allocated using a nested CET specification
GFTCapFix	Global free trade with fixed capital account
GFTLnd	Global free trade with upward sloping aggregate land supply curve
GFTFlxFF	Global free trade with upward sloping natural resource supply curve

Source: Author.

reflecting an increase of 0.18 percent. Most of the incremental improvement accrues to East Asia in both level and percent terms, with Oceania, South East Asia, Middle East & North Africa and Rest of the World seeing smaller but noticeable increases. North America, South Asia and Latin America, see small or negative changes to real income.

Table 6. Change in equivalent variation, \$million

Region	GFT	GFTCapFix	GFTCET	GFTFlxFF	GFTLnd
Oceania	3,102	2,071	1,056	1,291	3,430
East Asia	90,656	80,555	56,776	86,885	93,739
South East Asia	5,504	3,400	2,909	2,025	5,507
South Asia	-1,151	-1,125	-10,229	860	-1,254
North America	-6,947	3,813	2,166	-5,219	-3,503
Latin America	-2,934	-5,570	-6,435	-1,526	-1,109
European Union (28)	9,349	17,067	14,222	16,212	9,843
Mid. East & N. Afr.	5,731	3,312	-408	8,993	6,544
Sub-Saharan Africa	1,050	187	-3,358	3,297	1,644
Rest of the world	9,071	6,880	1,343	10,743	8,666
World	113,431	110,591	58,042	123,561	123,507

Source: Author.

Table 7. Change in equivalent variation, percent

Region	GFT	GFTCapFix	GFTCET	GFTFlxFF	GFTLnd
Oceania	0.23	0.15	0.08	0.09	0.25
East Asia	0.69	0.62	0.43	0.66	0.72
South East Asia	0.29	0.18	0.15	0.11	0.29
South Asia	-0.06	-0.05	-0.49	0.04	-0.06
North America	-0.04	0.02	0.01	-0.03	-0.02
Latin America	-0.07	-0.14	-0.16	-0.04	-0.03
European Union (28)	0.06	0.11	0.09	0.11	0.06
Mid. East & N. Afr.	0.16	0.09	-0.01	0.25	0.18
Sub-Saharan Africa	0.08	0.01	-0.25	0.25	0.12
Rest of the world	0.27	0.21	0.04	0.33	0.26
World	0.18	0.18	0.09	0.20	0.20

Source: Author.

The capital account closure has small impacts at the global level, however with some reallocation of the gains—mainly towards North America and the European Union. This is due to a lower increase in foreign savings accruing to developing countries (see Table 9), which under the default capital account see increases in

foreign savings as they become better investment opportunities for foreign capital. The CET specification for the disposition of domestic output across destination markets has the largest potential impact on global and regional welfare. The welfare gains globally are cut by nearly a half, with large relative reductions across developing regions, but gains for North America and the European Union. The CET specification reduces the ease with which countries can re-allocate their output to maximize their gains and also has impacts on the changes in the terms of trade (see Table 8). Appendix B provides analytical insights to this outcome. The loosening of supply constraints on land and natural resources allow for a modest expansion in output and increase the welfare gains. There is again some re-allocation of the global gains across regions reflecting in part their comparative advantage in resource-intensive production (and the user-supplied factor supply elasticities).

The terms of trade impacts across regions are highly differentiated using the standard model specification, Table 8. South Asia sees the largest negative impact, a reduction in the terms of trade of over 2 percent. In fact, there are negative terms of trade impacts for all regions with the exception of Oceania, East Asia and South East Asia. All regions benefit from lower import prices. However, regions with the highest tariffs see a large increase in import demand generated by the reduced tariffs—for example 10-11 percent in the case of South Asia and Sub-Saharan Africa. Even with some increase in net capital inflows, export revenues must rise to pay for the ex ante demand push for imports, leading to lower export prices to induce an increase in export demand. These mechanisms work across all model specifications. In the case of fixed capital flows, the export price decline increases modestly as there is a tightening of the balance of payments constraint. In the case of the CET specification, exporters have a reduced ability to take advantage of new opportunities, for example to export to countries that have significant tariff reductions. This raises their export prices in relative terms and reduces the demand for imports. This in turn reduces the need for raising exports for the tariff-reducing country and a lesser real-exchange rate depreciation.

Table 8. Change in the terms of trade, percent

Region	<i>GFT</i>	<i>GFTCapFix</i>	<i>GFTCET</i>	<i>GFTFlxFF</i>	<i>GFTLnd</i>
Oceania	0.42	0.24	0.00	0.18	0.34
East Asia	0.90	0.75	0.56	0.92	0.95
South East Asia	0.03	-0.10	-0.01	-0.03	0.04
South Asia	-2.14	-2.13	-1.06	-1.92	-2.19
North America	-0.17	0.10	0.31	-0.12	-0.22
Latin America	-0.86	-1.06	-0.99	-0.90	-0.98
European Union (28)	-0.06	0.05	0.12	0.01	-0.07
Mid. East & N. Afr.	-0.35	-0.49	-0.74	-0.56	-0.30
Sub-Saharan Africa	-0.78	-0.92	-1.19	-1.03	-0.76
Rest of the world	-0.37	-0.50	-0.80	-0.58	-0.34
World	-0.02	-0.02	0.02	-0.02	-0.02

Source: Author.

Most developing regions see an increase in the net inflow of savings, Table 9. Given the default specification, these regions see an ex ante increase in their rate of return and thus attract savings. This is financed by North America and the European Union. The total transfer in savings amounts to \$120 billion. Obviously the specification that fixes the capital account—the simulation labeled *GFTCapFix*—blocks this channel of economic re-structuring that leads to a somewhat stronger terms of trade impact.⁸¹ The CET specification dampens the flows slightly compared to the standard specification.

Table 9. Net savings inflow, percent of GDP

Region	<i>Base</i>	<i>GFT</i>	<i>GFTCapFix</i>	<i>GFTCET</i>	<i>GFTFlxFF</i>	<i>GFTLnd</i>
Oceania	-0.91	-0.63	-0.91	-0.80	-0.83	-0.60
East Asia	-2.66	-2.23	-2.61	-2.29	-2.29	-2.23
South East Asia	-2.69	-1.98	-2.68	-2.10	-2.10	-1.98
South Asia	9.11	9.42	9.35	9.38	9.42	9.42
North America	4.13	3.84	4.12	3.89	3.83	3.85
Latin America	-0.46	-0.19	-0.46	-0.34	-0.12	-0.14
European Union (28)	0.92	0.60	0.92	0.68	0.61	0.58
Mid. East & N. Afr.	-7.87	-7.76	-8.03	-7.78	-7.62	-7.77
Sub-Saharan Africa	-0.25	0.04	-0.26	0.00	0.18	0.04
Rest of the world	-8.40	-8.35	-8.53	-8.38	-8.20	-8.38
World	0.00	0.00	0.00	0.00	0.00	0.00

Source: Author.

A final table in this section, Table 10, highlights the impacts of introducing positive supply elasticities for land and natural resources. At the world level, the increase in natural resources from the free trade scenario is 0.6 percent relative to the base.⁸² The expansion is relatively strong in regions with a high elasticity, but the growth in supply will also depend on changing comparative advantage and thus we see the highest increase in Sub-Saharan Africa with supply growing by 4.5 percent. In a more disaggregated model, the supply elasticities would be differentiated across all activities that have a natural resource factor—fisheries, forestry, other mining, coal, oil and gas.

In the case of land, the global expansion is 0.8 percent, with expansion and contraction highly differentiated across regions.⁸³ The results suggest that tariffs on

⁸¹There are small deviations from the baseline as net saving flows are fixed in real terms and there are small valuation effects coming from the fact that the global investment price is used as the price deflator.

⁸²The elasticities are differentiated across regions. A high elasticity was assigned to Middle East & North Africa (4) and Rest of the world (3). A low elasticity (1) was assigned to East and South Asia and the European Union. All other regions had an intermediate value (2). The elasticities are entered in the simulation file prior to the calibration module.

⁸³A high elasticity (1.2) was assigned to Latin America and Sub-Saharan Africa—regions commonly assumed to have the most potential land for agricultural expansion. A low elasticity (0) was assigned to East and South Asia and the Middle East & North Africa. The elasticity was set to 0.6 for Oceania, North America and the European Union. A value of 0.3 was used for South East Asia. The

agriculture are relatively high in many regions as we see a large expansion of land in Oceania, and North and Latin America that take advantage of new market opportunities. The largest contractions occur in the European Union and Rest of the world.

Table 10. Change in factor supply, percent

Region	<i>GFTFlxFF</i>	<i>GFTLnd</i>
Oceania	-2.04	6.82
East Asia	-3.84	0.00
South East Asia	-4.17	-0.04
South Asia	0.50	0.00
North America	-0.23	7.23
Latin America	1.89	4.20
European Union (28)	1.43	-1.50
Mid. East & N. Afr.	2.15	0.00
Sub-Saharan Africa	4.53	1.50
Rest of the world	1.57	-3.32
World	0.61	0.80

Source: Author.

This section highlights potential impacts from some of the extensions built into the GAMS version of the GTAP model relative to the GEMPACK version. The aggregation of the GTAP Data Base used to highlight these extensions is quite stylized and the results should be perceived with a degree of caution. Nonetheless, they suggest that plausible and relatively simple extensions to the standard GTAP model can have substantive impacts on the results—though mostly in structural terms rather than in aggregate. One drawback to the CET specification is the lack of econometric estimates of the transformation elasticities. Perhaps the latter two simulations, i.e. the responsiveness of natural resources, are particularly relevant. History has shown that changes in the oil market can be a big driver of welfare changes with large impacts on the terms of trade and trade volumes.

7. Concluding remarks

This article presents a version of the standard GTAP model in GAMS. It complements other efforts, for example [Lanz and Rutherford \(2016\)](#). However, its main purpose is to provide an equivalent version of the GEMPACK-based model, albeit with some extensions. It has already proven to be useful to the Center for replication purposes as we attempt to understand differences in model results, even if relatively small numerically. This has led to changes in model specifications for both versions so that results are as similar as possible over a wide range of shocks. It can also be used as a starting point to develop various extensions of the GTAP model as described in detail in Section 4 in [Corong et al. \(2017\)](#) and put into practice, see [Britz and van der Mensbrughe \(2017\)](#). The latter includes variations of

elasticities were entered in the simulation file before the calibration module.

GTAP-E, GTAP-AEZ, and GTAP-HET as well as configurations that allow for comparisons with other well-known GTAP-based models such as MIRAGE, GLOBE and ENVISAGE. Many GAMS users may find it useful as a second entry point into the GTAP model and database. GEMPACK users are more than likely to be disappointed as it lacks many of the diagnostic tools that have been developed for the GEMPACK version of the GTAP model—including the extremely useful welfare decomposition tool (Huff and Hertel (2001) and Corong et al. (2017)).⁸⁴ In fact there is very limited post-processing of simulation results beyond the CSV cube created by the code and the GDX file that contains the full solution.⁸⁵ Despite some of these drawbacks, GEMPACK users may see some use as an entry point into the world of GAMS modeling. All modelers can learn from one another—not only in terms of model specification—but also in terms of what might be considered good practice, even if modeling remains as much of an art as it is a science.

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⁸⁴Note that Britz and van der Mensbrugghe (2017) provide an alternative welfare decomposition.

⁸⁵We have been developing somewhat more extensive tools for post-processing, including automated linkages to Excel Pivot tables and R scripts to produce graphs. We plan to make these publicly available in the near future.

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Appendix A. Model concordance for GAMS and GEMPACK Versions

The following three sets of tables provide the concordances for the variables and key parameters. The concordance links the variables as represented in this document with their GAMS and GEMPACK counterparts, i.e. the coded variables. In general there is a one-to-one correspondence. However there are exceptions that are described in the tables. Table A.1 provides the concordance for the endogenous variables. Table A.2 provides the concordance for the exogenous variables under standard model closure. Table A.3 provides the concordance for the key parameters such as CES substitution and CET transformation elasticities. A few additional notes:

- The GAMS code collapses all variables and parameters related to Armington demand and its decomposition into domestic and import components into a single set of variables and parameters indexed by *aa*. This reduces the size of the code considerably as only one equation is needed for all Armington agents instead of separate code for each agent—for example firms, private and public agents and investment. It has no impact on the model size or results. A '.' is used to express the Armington-related variables for the GEMPACK version, where the '.' is replaced by 'f' for firms, 'p' for private demand, 'g' for public demand and 'i' for investment demand.
- Most CET expressions in the GAMS code, for example the allocation of mobile factors across activities, recognize the value of 'INF' to signify perfect mobility. This reduces the number of equations relative to the GEMPACK code, which requires a separate expression for partial and full mobility assumptions. Again, this has no impact on the model size or results.
- In general, the variables as expressed in this document and in the GAMS code have the same name. Exceptions, often Greek symbols, are noted in the descriptive note.

Table A.1. Concordance of Endogenous Variables

GAMS	GEMPACK	GAMS Eqn.	GEMPACK Eqn.	Description
<i>ND</i>	<i>qint</i>	NDEQ	E_qint	Demand for aggregate intermediate bundle.
<i>VA</i>	<i>qva</i>	VAEQ	E_qva	Demand for aggregate value added bundle.
<i>PX</i>	<i>po</i>	PXEQ	E_qo	Unit cost, or zero profit condition. (N.B. The GEMPACK version reverses the names of E_qo and E_po.)
<i>XA</i>	<i>qfa</i>	XAPEQ	E_qfa	Intermediate demand. N.B. The GAMS version collapses all Armington demand into a single variable XA.
<i>PND</i>	<i>pint</i>	PNDEQ	E_pint	Price of aggregate intermediate demand bundle.
<i>XF</i>	<i>qfe</i>	XFEQ	E_qfe	Factor demand.
<i>PVA</i>	<i>pva</i>	PVAEQ	E_pva	Price of aggregate value added bundle.
<i>X</i>	<i>qca</i>	XEQ	E_qca	Supply of commodity <i>i</i> by activity <i>a</i> . (N.B. GEMPACK uses <i>c</i> to index commodities.)
<i>XP</i>	<i>qo</i>	XPEQ	E_po	Aggregate output of activity <i>a</i> .
<i>PP</i>	<i>ps</i>	PPEQ	E_ps	Tax-inclusive price of commodity <i>i</i> by supplied by activity <i>a</i> .
<i>P</i>	<i>pca</i>	PEQ	E_pca	Pre-tax price of commodity <i>i</i> by supplied by activity <i>a</i> .
<i>PS</i>	<i>pds</i>	PSEQ	E_qc	Supply price of commodity <i>i</i> .
<i>YTAX("PT")</i>	<i>del_taxrout</i>	YTAXEQ	E_del_taxrout	Tax revenues linked to the production tax. (N.B. The GEMPACK version calculates tax revenues relative to regional income.)
<i>YTAX("FT")</i>	<i>del_taxrfu</i>	YTAXEQ	E_del_taxrfu	Tax revenues linked to taxes on factor use.
<i>YTAX("FS")</i>		YTAXEQ		Tax revenues linked to subsidies on factor use. (N.B. Not part of the standard GEMPACK version.)
<i>YTAX("FC")</i>	<i>del_taxriu</i>	YTAXEQ	E_del_taxriu	Tax revenues linked to intermediate demand.
<i>YTAX("PC")</i>	<i>del_taxrpc</i>	YTAXEQ	E_del_taxrpc	Tax revenues linked to private consumption.
<i>YTAX("GC")</i>	<i>del_taxrgc</i>	YTAXEQ	E_del_taxrgc	Tax revenues linked to government consumption.
<i>YTAX("IC")</i>	<i>del_taxric</i>	YTAXEQ	E_del_taxric	Tax revenues linked to investment expenditures.
<i>YTAX("ET")</i>	<i>del_taxrexp</i>	YTAXEQ	E_del_taxrexp	Tax revenues linked to exports.
<i>YTAX("MT")</i>	<i>del_taxrimp</i>	YTAXEQ	E_del_taxrimp	Tax revenues linked to imports.
<i>YTAX("DT")</i>	<i>del_taxrinc</i>	YTAXEQ	E_del_taxrinc	Income tax revenues.
<i>YTaxTot</i>	<i>del_ttaxr</i>	YTaxTOTEQ	E_del_ttaxr	Total tax revenues.
<i>YTaxInd</i>	<i>del_indtaxr</i>	YTaxINDEQ	E_del_indtaxr	Total indirect tax revenues.
<i>factY</i>	<i>fincome</i>	FACTYEQ	E_fincome	Factor income net of depreciation.
<i>Y</i>	<i>y</i>	REGYEQ	E_y	Regional income. (N.B. In the GAMS code, the variable name is regY.)
<i>SAVE</i>	<i>qsave</i>	RSAVEQ	E_qsave	Nominal regional savings (coded as <i>rsav</i> in the GAMS code). (N.B. This variable is not explicit in the GEMPACK version and this equation determines the 'volume' of regional savings.)
<i>YG</i>	<i>yg</i>	YGEQ	E_yg	Nominal government expenditures.
<i>YC</i>	<i>yp</i>	YCEQ	E_yp	Nominal private expenditures.
Φ	<i>uelas</i>	PHIEQ	E_uelas	Elasticity of total expenditure with respect to utility (coded as <i>PHI</i> in the GAMS code).

Source: Author.

Table A.1 Concordance of Endogenous Variables, ctd.

GAMS	GEMPACK	GAMS Eqn.	GEMPACK Eqn.	Description
U	u	UEQ	E_u	Top level utility
U^S		USEQ		Utility of savings. (N.B. This variable is not explicit in the GEMPACK model.)
Z		ZCONSEQ		Auxiliary variable used to determine private demand (codes as ZCONS in the GAMS code). (N.B. Not used in the GEMPACK code.)
s^P		XCSHREQ		Private consumption budget shares. (N.B. Not part of the GEMPACK code though calculated as coefficient CONSHR in the solution process.)
XA	qpa	XACEQ	E_qpa	Private consumption. (N.B. The GAMS code collapses all of Armington demand into a single variable XA .)
ϕ^P	$uepriv$	PHIPEQ	E_uepriv	Elasticity of expenditure with respect to utility from private consumption (coded as phiP in the GAMS code).
PC	$ppriv$	PCONSEQ	E_ppriv	Private consumption price index (coded as PCONS in the GAMS code).
U^P	up	UHEQ	E_up	Utility of private consumption (coded as uh in the GAMS code).
XA	qga	XAGEQ	E_qga	Government consumption. (N.B. The GAMS code collapses all of Armington demand into a single variable XA .)
PG	$pgov$	PGEQ	E_pgov	Government expenditure price deflator.
U^G	ug	UGEQ	E_ug	Utility of public expenditure
XA	qia	XAIEQ	E_qia	Investment expenditures. (N.B. The GAMS code collapses all of Armington demand into a single variable XA .)
PI	$pinv$	PIEQ	E_pinv	Investment expenditure price deflator.
XI	$qinv$	XIEQ	E_qinv	Volume of investment.
PDP	$p.d$	PDPEQ	E_p.d	Tax-inclusive price of domestic goods. (N.B. GAMS collapses this into a single variable. In GEMPACK the relevant variables are pfd, ppd, pgd, and pid.)
PMP	$p.m$	PMPEQ	E_p.m	Tax-inclusive price of aggregate import goods. (N.B. GAMS collapses this into a single variable. In GEMPACK the relevant variables are pfm, ppm, pgm, and pim.)
PA	$p.a$	PAEQ	E_p.a	Agent-specific Armington price. (N.B. GAMS collapses this into a single variable. In GEMPACK the relevant variables are pfa, ppa, pga, and pia.)
XD	$q.d$	XDEQ	E_q.d	Agent-specific demand for domestic goods. (N.B. GAMS collapses this into a single variable. In GEMPACK the relevant variables are qfd, qpd, qgd and qid.)
XM	$q.m$	XMEQ	E_q.m	Agent-specific demand for aggregate import goods. (N.B. GAMS collapses this into a single variable. In GEMPACK the relevant variables are qfm, qpm, qgm and qim.)

Source: Author.

Table A.1 Concordance of Endogenous Variables, ctd.

GAMS	GEMPACK	GAMS Eqn.	GEMPACK Eqn.	Description
<i>XMT</i>	<i>qms</i>	XMTEQ	E_qms	Aggregate demand for imports.
<i>XW</i>	<i>qxs</i>	XWEQ	E_qxs	Bilateral demand for imports.
<i>PMT</i>	<i>pms</i>	PMTEQ	E_pms	Price of aggregate import bundle.
<i>XD^s</i>		XDSEQ		Supply of domestic goods. (N.B. The CET specification is not implemented in GEMPACK).
<i>XET</i>		XETEQ		Supply of aggregate exports. (N.B. The CET specification is not implemented in GEMPACK).
<i>XS</i>	<i>qds</i>	XSEQ	E_qds	Equilibrium condition for domestic goods. (N.B. In the GEMPACK code, this is derived assuming perfect transformation of domestic supply.)
<i>PE</i>		PEEQ		Bilateral producer price of exports. In the GEMPACK code, this is <i>pds</i> .
<i>PET</i>		PETEQ		Aggregate price of exports. (N.B. Not implemented in the GEMPACK code.)
<i>XWGM</i>		XWMGEQ		Total demand for trade margins to ship good <i>i</i> to region <i>s</i> to region <i>d</i> . (N.B. Substituted out in GEMPACK version.)
<i>XMGM</i>	<i>qtmfsd</i>	XMGMEQ	E_qtmfsd	Demand for bilateral trade margins by mode.
<i>PWGM</i>	<i>ptrans</i>	PWMGEQ	E_ptrans	Price of aggregate bilateral trade margin.
<i>XTMG</i>	<i>qtm</i>	XTMGEQ	E_qtm	Global demand for trade margins by mode.
<i>XA</i>	<i>qst</i>	XATMGEQ	E_qst	Regional supply of trade margins by mode. (N.B. In the GAMS version, domestic supply of margin services is treated as an Armington good, though given the initial data, it only demands domestic services.)
<i>PTMG</i>	<i>pt</i>	PTMGEQ	E_pt	Global price of margin services by mode.
<i>PE^{FOB}</i>	<i>pfob</i>	PEFOBEQ	E_pfob	Bilateral FOB export price.
<i>PM^{CIF}</i>	<i>pcif</i>	PMCIFEQ	E_pcif	Bilateral CIF import price.
<i>PM</i>	<i>pmds</i>	PMEQ	E_pmds	Bilateral import price tariff inclusive.
<i>PD</i>		PDEQ		Equilibrium price for domestic goods. (N.B. In GEMPACK, the price is <i>pds</i> as there is no difference by region of destination.)
<i>XFT</i>	<i>QE</i>	XFTEQ		Aggregate supply of mobile factors. (N.B. Aggregate supplies are fixed in the standard GTAP model.)
<i>PF</i>	<i>PEB</i>	PFEQ	E_qes1	Market price for factors. (In GEMPACK, this is split over three equations including E_qes2 and E_qes3 to handle the three different possible equilibrium conditions, i.e. perfect and partial factor mobility, and sector specific factors.)
<i>PFT</i>	<i>PE</i>	PFTEQ	E_pe1	Aggregate price of mobile factors.
<i>PFA</i>	<i>PFE</i>	PFABEQ	E_pfe	Purchaser's price of factors.
<i>PFY</i>	<i>PES</i>	PFYEQ	E_pes	Factor return net of income taxes.

Source: Author.

Table A.1 Concordance of Endogenous Variables, ctd.

GAMS	GEMPACK	GAMS Eqn.	GEMPACK Eqn.	Description
K^0	<i>kb</i>	KSTOCKEQ		Non-normalized level of the beginning of period capital stock, coded as KSTOCK in the GAMS code. (Equation not needed in the GEMPACK version.)
K^1	<i>ke</i>	KAPENDEQ	E_ke	End-of-period capital stock (coded as KAPEND in the GAMS code).
R^a	<i>rental</i>	ARENTEQ	E_rental	Gross return to capital net of tax (coded as ARENT in the GAMS code).
R^c	<i>rorc</i>	RORCEQ	E_rorc	Net return to capital after depreciation (coded as RORC in the GAMS code).
R^e	<i>rore</i>	ROREEQ	E_rore	Expected return to capital (coded as RORE in the GAMS code).
S^f	<i>del_fsave</i>	SAVFEQ	E_del_fsave	Net capital flows. (The GEMPACK version has additional equations depending on specific closures.)
R^g	<i>rorg</i>	RORGEQ	E_rorg	Global average expected return (different closures could impact which equations are deployed.)
\emptyset^f	<i>del_fsavery</i>	CHIFEQ	E_del_fsavery	Net foreign savings as a share of regional income.
YI	<i>vnetinv</i>	YIEQ	E_vnetinv	Gross investment (in value). (The equivalent variable in GEMPACK measures the value of net investment.)
XI_{GBL}	<i>globalcgds</i>	XIGBLEQ	E_globalcgds	Volume of global net investment.
PI_{GBL}	<i>pcgdswwld</i>	PIGBLEQ	E_pcgdswwld	Global average price of investment.
$PSave$	<i>psave</i>	PSAVEEQ	E_psave	Savings price.
$PABS$	<i>pabsFisher</i>	PABSEQ	E_pabsFisher	Domestic absorption Fisher price index.
$PMUV$	<i>pmuvFisher</i>	PMUVEQ	E_pmuvFisher	Fisher export price index of manufactured exports from developed regions.
$PFACT$	<i>pfactor</i>	PFACTEQ	E_pfactor	Fisher regional factor price index.
$PFACT^w$	<i>pfactwwld</i>	PFACTWEQ	E_pfactwwld	Fisher world factor price index.
$PNUM$		PNUMEQ		Model numéraire. (In GEMPACK, model numéraire can be changed with a swap.)

Source: Author.

Table A.2. Concordance of Exogenous variables

Symbol	GAMS	GEMPACK	Description
τ^p	PRDTX	t o	Production tax.
τ^{dtx}	DINTX	t . d	Indirect tax on sales of domestic goods. (N.B. The GAMS version consolidates this tax for all Armington agents. In GEMPACK the relevant variables are t f d, t p d, t g d, and t i d.)
τ^{mtx}	MINTX	t . m	Indirect tax on sales of import goods. (N.B. The GAMS version consolidates this tax for all Armington agents. In GEMPACK the relevant variables are t f m, t p m, t g m, and t i m.)
τ^{ft}	FCTTX	t f e	Tax on factor use.
τ^{fs}	FCTTS		Subsidy on factor use. (N.B. There is no equivalent in the standard GEMPACK version.)
τ^m	IMPTX	t m s	Bilateral import tariff.
τ^e	EXPTX	t x s	Bilateral export tax.
κ^f	KAPPAF	t i n c	Tax on factor income—by factor and source activity.
Pop	POP	p o p	Total population.
ζ^{ms}	TMARG		Trade margin wedge. (N.B. This is not needed in the GEMPACK version.)
XFT	XFT	q e	Aggregate factor supply. (N.B. This is only exogenous in the standard GEMPACK version. In the GAMS version, it is endogenous, even if the supply elasticity is at its default value of 0.)
XF^s	XF	q f e	Supply of sector-specific factors. (N.B. This is only exogenous in the standard GEMPACK model.)

Source: Author.

Table A.3. Concordance of Key Parameters

Symbol	GAMS	GEMPACK	Description
σ^p	SIGMAP	ESUBT	Top level production nest CES elasticity (<i>ND</i> vs. <i>VA</i>)
σ^{nd}	SIGMAND	ESUBC	CES elasticity across intermediate inputs
σ^{na}	SIGMAVA	ESUBVA	CES elasticity across factors of production
ω^s	OMEGAS	ETRAQ	Make transformation elasticity.
σ^s	SIGMAS	ESUBQ	Make substitution elasticity. In the case of GEMPACK, it is entered as the inverse to allow for perfect substitution, i.e. a value of zero implies perfect substitution. The GEMPACK version does not allow for Leontief preferences. The GAMS version recognizes 'INF' to indicate perfect substitution.
σ^m	SIGMAM	ESUBD	Top level Armington elasticity. In GAMS, it is indexed by Armington agent. The GEMPACK version assumes uniformity across all agents.
σ^{wv}	SIGMAW	ESUBM	Second level Armington elasticity, i.e. across imports by source.
e^{RoR}	RORFLEX	RORFLEX	Elasticity of the expected rate of return to the growth of capital.
b	BH	SUBPAR	Substitution parameter in the CDE function.
e	EH	INCPAR	Expansion parameter in the CDE function.
ω^f	OMEGAF	ETRAE	Transformation elasticity for mobile factors. (N.B. The GAMS version uses 'INF' for perfect transformation.)
σ^i	SIGMAI	ESUBI	CES substitution elasticity in investment expenditures.
σ^g	SIGMAG	ESUBG	CES substitution elasticity in government expenditures.
σ^{mg}	SIGMAMG	ESUBS	CES substitution elasticity in demand for margin services.
ω^x	OMEGAX		Top level CET transformation elasticity, i.e. between domestic market and aggregate export market. The default value is infinity—the implicit assumption in GEMPACK.
ω^{wv}	OMEGAW		Second level CET transformation elasticity, i.e. across destination markets. The default value is infinity—the implicit assumption in GEMPACK.
η^{ft}	ETAFF		Supply elasticity for mobile factors. In the standard GEMPACK version, supply elasticities are implicitly zero.
η^{ff}	ETAFF		Supply elasticity for sector-specific factors (i.e. natural resources). In the standard GEMPACK version, supply elasticities are zero.

Source: Author.

Appendix B. CET Specification for the allocation of domestic output

Many single country models and some global trade models implement imperfect transformation of domestic output by destination, i.e. between domestic and export markets. This is a core feature of the 1-2-3 model described in [Devarajan et al. \(1990\)](#), and also a feature of IFPRI's standard model ([Lofgren et al. \(2002\)](#)). Both of these models are single country models with a single export destination. The CET was implemented in a global model as early as 1989 in the OECD's Walras Model ([van der Mensbrugge et al. \(1990\)](#)) and incorporated in both of the World Bank's global CGE models: LINKAGE and ENVISAGE. In the case of these latter two models, the CET was implemented as an optional feature with both models allowing for perfect transformation. In global models, the CET is typically implemented using a nested structure, analogous to the nested Armington for import demand. A top level CET nest allocates total output between the domestic market and the aggregate export market. A subsequent nest allocates aggregate exports across all foreign destination markets.

This section provides some analytical insights from implementing the CET specification in the case of a small economy with fixed terms of trade. This is the same model developed in [Devarajan et al. \(1990\)](#). The model is given in the following set of equations:

<i>Equation</i>	<i>Description</i>
$XD = \gamma^d X \left(\frac{PD}{P} \right)^\omega$	Supply of domestic goods
$XE = \gamma^e X \left(\frac{PE}{P} \right)^\omega$	Supply of export goods
$P = \left[\gamma^d PD^{1+\omega} + \gamma^e PE^{1+\omega} \right]^{1/(1+\omega)}$	Output price
$PA \cdot XA = P \cdot X + \tau^m \cdot EXR \cdot PM^w \cdot XM + EXR \cdot Sf$	Domestic absorption
$XD = \alpha^d XA \left(\frac{PA}{PD} \right)^\sigma$	Price of domestic goods
$XM = \alpha^m XA \left(\frac{PA}{PM} \right)^\sigma$	Demand for imported goods
$PA = \left[\alpha^d PD^{1-\sigma} + \alpha^m PM^{1-\sigma} \right]^{1/(1-\sigma)}$	Armington price
$PM = (1 + \tau^m) EXR \cdot PE^w$	Domestic price of imports
$PE = EXR \cdot PE^w$	Domestic price of exports

The first three set of equations implement the CET specification, where ω is the transformation elasticity. Output, X , is assumed fixed in this simple macro model.⁸⁶ In the case of perfect transformation, the default value in the GTAP

⁸⁶It would be easy to add a production function, for example a CES as a function of labor and

model, the equations are replaced with $PD = P$, $PE = P$, and $X = XD + XE$. The fourth equation represents national income—the sum of the value of output, tariff revenues and foreign inflows (assumed to be positive herein). The next three set of equations implement Armington demand for domestic and import goods, where σ is the substitution elasticity.⁸⁷ The last two equations convert world trade prices into domestic prices, with import prices augmented by the tariff.

In the canonical model the small country assumption holds and thus PM^w and PE^w are exogenous. We also assume a fixed current account, i.e. S^f is exogenous and the only policy instrument is the import tariff, τ^m . The model numéraire is the exchange rate, EXR , and is fixed at 1. The left out equation, i.e. Walras' Law, is the trade balance: $PM^w XM = PE^w XE + S^f$. We will log-differentiate the model holding all exogenous variables fixed, but varying the import tariff. Using the standard practice in GEMPACK models, we will express the percent changes relative to the percent change in the power of the tariff, defined as $T = (1 + \tau^m)$.⁸⁸

Table B.1 provides the log-linearized equations with a focus on the role of the CET elasticity. There are two extreme values for the transformation elasticity. The first is 0, in which case domestic output volume shares are fixed, i.e. there is no transformation possibility. At the other extreme is perfect transformation, when the law of one price must hold for domestic and export markets.

The impact of the CET elasticity is captured in the β parameter defined as:⁸⁹

$$\beta = \frac{\sigma}{\sigma + (1 - r^d s^f)\omega}$$

Both extreme values for the transformation elasticity translate into values for the β parameter. β ranges from 1 for zero transformation to 0 for perfect transformation. In the absence of foreign income flows, the formula for β simplifies to $\beta = \sigma / (\sigma + \omega)$.

When ω is equal to zero, the only changes to the equilibrium solution is an identical change to demand prices, with no change in volumes. There is no volume flexibility in this economy. The other extreme holds for perfect transformation. In this case the law-of-one-price must hold for the price of domestic goods, which are then pegged to the exogenous export price. With a reduction in the tariff, consumers switch to import goods and XM rises unambiguously. Demand for domestic goods always falls as tariffs increase. The condition for rising demand for domestic goods requires $\lambda^t > s^m$, which is impossible for any positive tariff.

capital. If factors are fixed, the first order conditions simply determine factor returns.

⁸⁷We have substituted out the equilibrium condition for domestic goods, XD , and thus this equation implicitly defines PD .

⁸⁸We can show that the following relation holds: $\dot{t} = \dot{\tau} \frac{\tau}{T}$.

⁸⁹The coefficients r^d and s^f are defined below. The former is the share of domestic sales in domestic output and the latter is the share of foreign inflows relative to imports and assumed positive.

Exports rise in step with imports, albeit adjusted by the share of export revenues in the balance of payment. If the foreign saving share is zero, then exports rise at the same rate as imports. Welfare gains are unambiguously positive as domestic absorption increases as β approaches zero, i.e. as we converge towards perfect transformation.

Table B.1. Log-linearized 1-2-3 Model

Variable	$\omega = 0$	$0 < \omega < \infty$	$\omega = \infty$
	$\beta = 1$	$0 < \beta < 1$	$\beta = 0$
$\dot{p}d$	$= \dot{t}$	$\beta \cdot \dot{t}$	0
$\dot{p}a$	$= \dot{t}$	$[\beta + s^m (1 - \beta)] \dot{t}$	$s^m \cdot \dot{t}$
\dot{p}	$= r^d \cdot \dot{t}$	$r^d \cdot \beta \cdot \dot{t}$	0
$\dot{x}d$	$= 0$	$(\beta - 1) \left(\frac{\lambda^t - s^m}{1 - \lambda^t} \right) \sigma \cdot \dot{t}$	$-\left(\frac{\lambda^t - s^m}{1 - \lambda^t} \right) \sigma \cdot \dot{t}$
$\dot{x}m$	$= 0$	$(\beta - 1) \left(\frac{s^d}{1 - \lambda^t} \right) \sigma \cdot \dot{t}$	$-\left(\frac{s^d}{1 - \lambda^t} \right) \sigma \cdot \dot{t}$
$\dot{x}a$	$= 0$	$(\beta - 1) \frac{\lambda^t}{1 - \lambda^t} s^d \cdot \sigma \cdot \dot{t}$	$-\frac{\lambda^t}{1 - \lambda^t} s^d \cdot \sigma \cdot \dot{t}$
$\dot{x}e$	$= 0$	$\frac{(\beta - 1)}{s^e} \left(\frac{s^d}{1 - \lambda^t} \right) \sigma \cdot \dot{t}$	$-\frac{1}{s^e} \left(\frac{s^d}{1 - \lambda^t} \right) \sigma \cdot \dot{t}$

Source: Author.

The log-linearized version of the model relies on a set of share coefficients described below.

$$s^d = \frac{PD \cdot XD}{PA \cdot XA} \quad s^m = \frac{PM \cdot XM}{PA \cdot XA} \quad \lambda^t = \frac{\tau^m \cdot EXR \cdot PWM \cdot XM}{PA \cdot XA}$$

$$r^d = \frac{PD \cdot XD}{P \cdot X} \quad s^e = \frac{PE^w \cdot XE}{PM^w \cdot XM} \quad s^f = \frac{S^f}{PM^w \cdot XM}$$

The implication of this derivation of the 1-2-3 model is that the elasticity of the change in volumes relative to the CET elasticity is equal to β . For example, if σ is 2 and ω is 4, then β equals 1/3, and the increase in import demand will therefore be 1/3 lower than in the case of perfect transformation (assuming no foreign inflows). As σ rises with respect to ω , the CET markedly dampens the responsiveness of the economy as β in this case converges towards 1.

Appendix C. Model Implementation

This appendix describes the implementation of the standard GTAP model in GAMS.

C.1 Data

The data input for the model is an aggregated database using GTAPAgg2—a publicly available aggregation facility used to aggregate the GTAP Data Base. GTAPAgg2 produces an aggregated database and its associated parameter file and sets file in the so-called 'HAR' format that is used by the GEMPACK suite of tools. One useful tool is the program 'HAR2GDX' that converts 'HAR' files to the 'GDX' format, which is the required format for the GAMS version of GTAP. Thus data preparation is a two-step procedure: 1) Create an aggregation using GTAPAgg2; and 2) convert the resulting 'HAR' files to 'GDX' format using 'HAR2GDX'. The model requires at least three files: the sets definition, the base data and the default parameter values. The GAMS code assumes that the names of all files start with the same code, for example 9x10, and all have a 'GDX' extension. The user defines `BaseName` as part of the simulation file and the code will then read from `BaseNameSets.gdx`, `BaseNameDat.gdx` and `BaseNamePrm.gdx`. The latter often has the name `default.prm` when emerging from the GEMPACK aggregation facility. The model will optionally read the CO₂ emissions file that is expected to have the name `BaseNameEmiss.gdx`. The software comes with a small command file, `convert.cmd`, which provides an example of how to automate the conversion from 'HAR' to 'GDX'.⁹⁰

C.2 Code

The core code is split into seven GAMS file—intended to provide some degree of code modularity. These files can be modified by the user, but are otherwise standalone, i.e. they need no revision to run the standard version of the model. The only file that users would need to change is the simulation file itself, which is described in the next section. The following describes each of the model components:

- 1) **getData.gms** This file reads the sets, data and parameters for the model. These are read in from the GDX files that emerge from the aggregation and conversion of the aggregated 'HAR' files to 'GDX' format. The model is

⁹⁰The CGEBox version of the model Britz and van der Mensbrugghe (2017) integrates two additional pre-simulation processes. Similar to Rutherford's GTAPinGAMS package (Lanz and Rutherford (2016)), the CGEBox version includes the so-called filtering program, which removes small flows with little or no influence on simulation results. This can reduce significantly the size of models and also render them more stable numerically (see Britz and van der Mensbrugghe (2016)). The second pre-simulation process is a version of *Altntax* that allows for changes to the benchmark database, for example tax rates (see Malcolm (1998)). We are working on a similar implementation with this standalone version of the standard GTAP model in GAMS.

equipped to handle CO₂ emissions and will read the emissions database if it exists. In addition, users have the ability to override default parameters at a later stage before model calibration.

- 2) **model.gms** This file contains the full model specification including the declaration of model parameters, variables and equations and definition of the equations and the model. The model equations are listed in the same order as the model description provided herein, which allows for a relatively transparent comparison of the mathematical description and model expression. The code also provides references to the relevant equations in the GEMPACK version of the model—both for GTAP ‘classic’ as well as the new standard version of the model.
- 3) **cal.gms** This file contains the code that initializes the model variables and calibrates all parameters. Variable initialization typically assumes that basic prices are set to 1, tax rates are calculated using the relevant data matrices of the values of flows, and volumes are initialized given the values, price initialization, and, where relevant, tax rates. Users provide key model parameters—mostly substitution elasticities and price and income elasticities for consumer demand. Most other parameters are calibrated, such as CES share parameters, in order to re-produce the benchmark data. Given the structure of the model, calibration can proceed recursively on a block by block basis. Note that the last few lines of the calibration code re-scale production variables. This is intended to improve the numerical properties of the model. Users should take care in using post-simulations results and ensure that the relevant production related variables are re-scaled back to their appropriate level.
- 4) **iterloop.gms**. This file contains code that takes care of preparing for a simulation between periods. It deals with closure, exogenizing inactive variables, fixing lags and setting bounds. It is also useful in a dynamic setting as it initializes variables in period ‘t’ with the relevant solution from period ‘t-1’. All variables and equations are time sub-scripted and initialized to benchmark levels. For comparative static simulations this is normally sufficient to have good numerical convergence. In a dynamic setting, the benchmark levels are not an appropriate initialization for period ‘t’.⁹¹
- 5) **mvar.gms** The file is linked to ‘iterloop.gms’ and is used to initialize variables at the beginning of each simulation.
- 6) **solve.gms** This file invokes the appropriate solver and updates variables

⁹¹This is the main drawback of coding the model with a time index. An alternative would be to drop the time index from all variables and equations then the solution for period ‘t-1’ automatically becomes the initial solution for period ‘t’. Additional code is then necessary to save the solution for ‘t-1’ to auxiliary parameters. This slight inconvenience is out-weighed by the convenience of the time index and the (remote) possibility of converting the model to full time consistency with forward looking behavior.

that may have been substituted out of the model (to limit its size).

- 7) **postsim.gms** This file performs various calculations using model results—including evaluating each region’s social accounting matrix (SAM). At user request, it will also save a number of model results into a CSV file. A number of different software packages can load CSV files including Excel—preferably into a Pivot table—and R.

C.3 Simulation file

This section describes the simulation file—this is the only file that a user needs to modify in order to run a simulation, i.e. the core files need no user intervention unless the user decides to modify the model specification.

The simulation file is divided into several parts. The first part sets a number of overarching elements for defining the simulation. Listing C.2 is an example of a simulation file and is used to run standard diagnostic checks on the model.⁹² The supplemental materials also includes an additional simulation file, *gft.gms*, that eliminates import tariffs globally, along with the files that are used to assess global free trade under alternative model specifications.

Table C.1 describes the first set of global options that need to be provided. The following section in the listing, lines 19–36, are linked to the ‘time’ dimension of the model. For comparative static simulations this normally involves three distinct ‘time’ periods, though could be as short as two and greater than three. For diagnostic purposes, it is standard to have three time periods. The first is called the *base* and it simply represents model initialization in the absence of a model solution. The second is called *check*. It solves the model with no shock. Except for small numerical deviations, it should exactly re-produce the *base*. One could verify this by looking at the output results. There are two additional indicators. The first is to have the listing file contain the initialized equations.⁹³ A search for *LHS* in the listing file will indicate any equations that have a significant deviation between the left- and right-hand side of the initialized equation—also sometimes called a residual check. A significant deviation is indicated by ‘*****’.⁹⁴ The second indicator is the maximum of the residuals, which is displayed in the listing file as *MAXIMUM OF F*. This should also be close to zero and typically reflects the largest accounting discrepancy in the input database (appropriately scaled). In the example listing file, the third period, identified as *shock*, raises the numéraire by 50% (see lines 160–162). If the model is correctly specified as homogeneous in prices, this shock should raise all prices and values by 50% and leave the volumes unchanged. In addition to the overall time frame, the user should identify the base year (*t0*). The

⁹²The term *listing* simply refers to the GAMS code and not to the output listing produced by the GAMS software.

⁹³This involves setting the *limrow* option to a positive number. By default, it is 3.

⁹⁴The tolerance can be controlled by setting the *tolinfrep* option for the model.

remaining elements of the time framework require no user inputs.

Table C.1. Global simulation options

Option	Description
simType	The model has a somewhat different configuration for comparative static and dynamic simulations. This option should be set to <code>CompStat</code> for comparative static simulations and <code>RcvDyn</code> for dynamic simulations.
simName	This is normally a short code, for example <code>Comp</code> , that is used to identify each simulation. Its main purpose is to provide a basename for the simulation output files.
baseName	This identifies the base for the input files, for example <code>9x10</code> . All input file names will be identified by the same base with an identifier, for example <code>sets</code> for the file containing the set definitions.
inDir	This identifies the location of the directory of the input files. If the input files are in the same directory as the simulation file, use <code>'.'</code> , which indicates use of the current directory. It should not include a closing backslash.
outDir	This identifies the location of the directory for the output files. Similar to the input directory, a value of <code>'.'</code> will save output to the current directory.
utility	Users must indicate the desired form of the utility function. Only two options are available: <code>CDE</code> for the CDE utility function, and <code>CD</code> for the Cobb-Douglas utility function. The latter is intended for implementing the Alvertax procedure.
savfFlag	The model has four possible closures for the capital account. (1) <code>CapFlex</code> allows for capital flows to react to changes in the expected relative rates of return across regions. (2) <code>CapFixShr</code> assumes the allocation of global investment across regions is the same as in the base. (3) <code>CapFix</code> assumes the capital account (in real terms) is exogenous. By default the allocation will be the same as in the base, but users can modify the allocation (using the <code>savfBar</code> parameter). In principle, the exogenous allocation should add to 0 globally, but this is ensured by the model by having a residual lender/borrower. (4) <code>CapSFix</code> fixes the capital account for each region relative to regional income. The ratio is calibrated to base year levels, but can be changed by the user. Similar to the <code>capFix</code> closure, there is a residual region that guarantees global adding up.
ifCAL	This parameter is intended for dynamic models. If it is set to 1, some economy-wide variable, for example labor productivity, will be used as an instrument to achieve a targeted growth for per capita GDP. If it is set to 0, the economywide variable is exogenous and GDP is endogenous.
ifSUB	Setting this parameter to 1 will implement the model with substitution. This reduces significantly the size of the model as many variables are substituted out by simple linear expressions of other model variables. The variables are updated at the end of each simulation.

Source: Author.

The next set of global options are identified in Table C.2 and correspond to lines 40–54 in the listing file. The first three are initialized automatically as they rely on previous settings.

The subsequent section initializes the CSV output files (see lines 58–84)—the first will hold the simulation results, and the second, if used, will hold user-based diagnostic information.

This is followed by the reading of the input files—sets (to provide the model dimensions), data (for model initialization) and the parameters. The `getData.gms` file is invoked for this purpose. This file will also initialize a number of elasticities that are part of the GAMS version of the model, but not the GEMPACK version—for example the factor supply elasticities. They are initialized to their default values, but can subsequently be overridden by the user (see below). After the model

Table C.2. Additional global simulation options

Option	Description
<code>ifDebug</code>	This option can be used to output intermediate results—for example in the calibration code. It is mainly used for diagnostics. In addition, it can be used to test the validity of calibration and initialization as it randomly initializes some of the core prices—instead of initializing them at 1.
<code>inScale</code>	The input database is evaluated in millions of dollars (of the reference year). A good rule of thumb for GAMS is to have GDP in the range of 10 to 100, thus using a scale of a million has the model evaluated in trillions of dollars, where world GDP is around 100.
<code>xpScale</code>	Despite re-scaling the model using the global scale parameter <code>inScale</code> , there can still be significant differences in orders of magnitudes across regions and sectors. It has proven useful to scale output vectors by a region and sector specific scalar (see parameter <code>xScale</code>). This parameter, <code>xpScale</code> , gives an order of magnitude for the output scale. A value of 1 will scale all output vectors so that output is between 1 and 10. The internal scale parameter, <code>xScale</code> , is always a power of 10.
<code>ifCSV</code>	One of the output files is a CSV file that can get quite large even for relatively aggregated models. This option, if set to 0, will shut off the writing of output to the CSV file.
<code>ifCSVAppend</code>	It is possible to save output from multiple simulations in the same CSV file if this parameter is set to 1.
<code>ifMCP</code>	This parameter governs the choice of solver. If it is set to 1, the PATH solver will be invoked. A setting of 0 will invoke the default NLP solver. In the latter case, the model will attempt to maximize the value of <i>Walras</i> , which should evaluate to (near) 0 if the model is correctly specified.

Source: Author.

dimensions are known, the user must add some additional sets required for the model (see lines 100-119). The model needs a definition of the labor subset (`l`) of the factors of production. This is mainly used for the dynamic version of the model where labor bias productivity is invoked. It also requires a definition of the capital subset (`cap`). A residual region (`rres`) is needed for two purposes. The first is to identify the left-out (investment) equation for Walras' law. The second is for the residual borrower/lender under some of the capital account closure rules. Note that the first reason has no impact on the model results—as any investment equation could be dropped. The second could lead to (presumably modest) differences in model results as this choice will impact the endogenous capital account of the residual region. The final user-defined subsets are used to specify exporting regions and commodities that define the so-called *MUV* index, which is intended to reflect the World Bank's so-called Manufacturing Unit Value indicator. In the model this is represented by the variable `PMUV`. This could be made the model numéraire, instead of the default `pwfact`. The exporting regions are typically assumed to be the high-income or OECD regions. The included commodities are typically manufactures.

The next section allows for parameter overrides. There are none defined in the example listing file. This is where one could change parameters such as the factor supply elasticities, the output (CET) transformation elasticities, etc. The overrides should come before the calibration code.

This is followed by model definition and initialization (lines 129–141). This sec-

tion loads two files—`model.gms` and `cal.gms`.

The next section is the core of the simulation file. It loops over all time periods and solves the model—with the exception of the first time period. At the beginning of each time period, the GAMS file `iterloop.gms` is loaded, which contains code that initializes the simulation at the beginning of each time period. The subsets `rs` and `ts` control which equations are included in the model. For the global model, `rs` should be set to `yes` for all regions.⁹⁵ This version is currently only designed to run as a globally linked model. The second key subset is `ts`. All equations are indexed by time. The subset `ts` controls which periods are involved in a solution. For the recursive dynamic version, only one period is solved for at a time and thus `ts` is only active for one year at a time.⁹⁶

The user shock is normally inserted between `iterloop.gms` and the solve statement. In the example file, the only user shock is the numéraire shock (see lines 160–162).

After setting some additional GAMS options the solver is invoked (see lines 166–184.), for all time periods save the first. There are different versions of the model for the comparative static and dynamic setups. When all time loops have been solved, the model saves the results—in a CSV file optionally, and as a GDX file.

The simulation file can be run from the GAMS IDE or in a command window.⁹⁷ In the case of the latter, it can be invoked by typing:

```
gams comp -pw=132
```

Large shocks may pose problems for the GAMS solver and it is sometimes necessary to break them into smaller steps. For example, the model fails to converge when setting all tariffs to zero, and attempting to solve in one-step. The file `GFT.GMS` is an example of how to cut shocks into smaller steps and improve convergence and listing C.1 provides a simplified snippet of the code. The shock sets all import tariffs to zero. The algorithm loops over a fixed number of iterations, where the import taxes linearly decline to zero. In the file the number of iterations is 10.⁹⁸ Note that the iteration loops over all 10 steps, but only invokes the solver

⁹⁵The model has been tested for use as a single region model, mainly to iterate recursively over each individual region before solving the global model. The purpose of this is to help with convergence—both numerically and for speed. See Britz and van der Mensbrugghe (2016).

⁹⁶With some additional effort, the model could be made to solve for all time periods simultaneously, but with little value in the absence of forward looking behavior.

⁹⁷This version of the code has been tested with GAMS Version 29 and above. There is at least one feature that may not be compatible with earlier GAMS versions and this is the use of '\$\$' for GAMS' dollar control options. Since 2012, with Version 24.0, GAMS introduced the '\$\$' option for dollar control options, which allows these options to be indented. Indentation improves code readability. Pre-24 releases require the single dollar symbol to be in the first column. See also https://www.gams.com/latest/docs/UG_DollarControlOptions.html.

⁹⁸If the model fails to converge users should attempt to set the number of steps to a greater level.

for 9 of them. The tenth will be solved by the regular solve step when taxes are set to zero.

Listing C.1. Solving through iteration

```

1  set iter / it1*it10 / ;
3  loop(tsim,
4    ts(tsim) = yes ;
6    $$include "iterloop.gms"
8    * Phase out import taxes
10   if(sameas(tsim, "shock"),
11     loop(iter,
12       imptx.fx(s,i,d,tsim) = (1-ord(iter)/card(iter))*imptx.l(s,i,d,tsim-1) ;
13       if(ord(iter) lt card(iter),
14         $$batinclude "solve.gms" gtap
15       ) ;
16     ) ;
17   ) ;
19   if(years(tsim) gt firstYear,
20     $$batinclude "solve.gms" gtap
21   ) ;
23   ts(tsim) = no ;
24 ) ;

```

In summary the user typically only needs to modify an existing simulation file. For each new database (or aggregation) the user needs to provide the `baseName`, and user definitions for labor, capital, the residual region, and the regions and commodities that will be used for defining the *PMUV* price index. The user may also decide to change key parameter values instead of using the default values. For each new simulation the user should provide a brief description of the simulation at the top of the file, a new simulation code (`simName`), optionally new names for the time period(s) and implementation of the shock(s). If the shocks are large, the simulation file may need some additional code to cut the shocks into smaller steps.

The supplemental materials include all of the simulation files: `COMP.GMS` for diagnostic testing of the model, `GFT.GMS` the free trade scenario using the standard model, `GFTCET.GMS` the free trade scenario with the CET specification for the allocation of domestic production across destination markets, `GFTCAPFix.GMS` the free trade scenario with fixed foreign savings, `GFTFlxFF.GMS` the free trade scenario with upward sloping supply curves for natural resources and `GFTLnd.GMS` the free trade scenario with upward sloping aggregate land supply curves. In addition to the simulation files, the supplemental material includes two Windows 'CMD' files that automate running the simulations and extracting key model indicators and merging them in a single CSV cube. The file `runall.cmd` runs all

This is done by changing the set `iter` located on line 152 in the distribution file.

of the simulations in sequence and then runs the command file `merge.cmd`. The file `merge.cmd` runs a GAMS program called `merge.gms` that extracts a handful of indicators from the simulation results and merges them into a single CSV cube. The latter can be imported into an Excel pivot table or loaded into R to facilitate the production of charts. It would be relatively easy to extend `merge.gms` to include additional indicators.

Listing C.2. Sample simulation file

```
1 * -----
2 *
3 * Standard model diagnostics
4 *
5 * Model preamble -- user options
6 *
7 * -----
8
9 $setGlobal simType CompStat
10 $setGlobal simName COMP
11 $setGlobal baseName 10x10
12 $setGlobal inDir .
13 $setGlobal outDir .
14 $setGlobal utility cde
15 $setGlobal savfFlag capFix
16 $setGlobal ifCal 0
17 $setGlobal ifSUB 1
18
19 set
20     t      "Time frame" / base, check, shock /
21     t0(t)  "Base year" / base /
22     ts(t)  "Time flag"
23 ;
24 alias(t,tsim) ;
25
26 Parameter
27     years(t)
28     gap(t)
29     FirstYear
30 ;
31
32 years(t) = ord(t) ;
33 gap(t) = 1 ;
34 loop(t0,
35     FirstYear = years(t0) ;
36 ) ;
37
38 ts(t) = no ;
39
40 scalar
41     ifSUB    "Set to 1 to reduce model size" / %ifSUB% /
42     ifCal    "Set to 1 to calibrate dynamically" / %ifCal% /
43     $$iftheni "%simType%" == "CompStat"
44         ifDyn    "Set to 1 to for a dynamic scenario" / 0 /
45     $$else
46         ifDyn    "Set to 1 to for a dynamic scenario" / 1 /
47     $$endif
48     ifDebug  "Set to 1 to debug calibration" / 0 /
49     inScale  "Scale for input data" / 1e-6 /
50     xpScale  "Scale factor for output" / 1 /
51     ifCSV    "Flag for CSV file" / 1 /
52     ifCSVAppend "Flag to append to existing CSV file" / 0 /
53     ifMCP    "Set to 1 to solve using MCP" / 1 /
54 ;
55
56 * CSV results go to this file
57
58 file
```

```
59  csv    / "%outDir%/%simName%.csv" /
60  screen / con /
61  ;

63  if(ifCSV,
64    if(ifCSVAppend,
65      csv.ap = 1 ;
66      put csv ;
67    else
68      csv.ap = 0 ;
69      put csv ;
70      put "Variable,Region,Sector,Qualifier,Year,Value" / ;
71    ) ;
72  csv.pc=5 ;
73  csv.nd=9 ;
74  ) ;

76  * This file is optional--sometimes useful to debug model

78  file debug / "%outDir%/%simName%DBG.csv" / ;
79  if(ifDebug,
80    put debug ;
81    put "Var,Region,Sector,Qual,Year,Value" / ;
82    debug.pc=5 ;
83    debug.nd=9 ;
84  ) ;

86  * -----
87  *
88  * Retrieve GTAP sets, data and parameters
89  *
90  * -----

92  $include "getData.gms"

94  * -----
95  *
96  * Required user definitions
97  *
98  * -----

100 set l(fp) "Labor factors" /
101     UnSkLab      "Unskilled labor"
102     SkLab        "Skilled labor"
103 / ;

105 set cap(fp) "Capital factor" /
106     Capital
107 / ;

109 set rres(r) "Residual region" /
110     NAmerica
111 / ;

113 set rmuv(r) "RMUV regions" /
114     Oceania, NAmerica, EU_28
115 / ;

117 set imuv(i) "IMUV commodities" /
118     c_procfood, c_textWapp, c_LightMnfc, c_HeavyMnfc
```

```
119 / ;
121 * -----
122 *
123 * Parameter overrides, for example factor supply elasticities,
124 *   output transformation elasticities
125 *
126 * -----

129 * -----
130 *
131 * Load model, initialize variables and calibrate parameters
132 *
133 * -----

135 * Get the model specification

137 $include "model.gms"

139 * Initialize the model

141 $include "cal.gms"

143 * -----
144 *
145 * Run the simulations for each time period
146 *
147 * -----

149 rs(r) = yes ;
150 ts(t) = no ;

152 loop(tsim,

154   ts(tsim) = yes ;

156   $$include "iterloop.gms"

158 * Define the simulation specific shock

160   if(sameas(tsim, "shock"),
161     pnum.fx(tsim) = 1.5 ;
162   ) ;

164   options limrow = 3, limcol = 3, solprint = off, iterlim = 1000 ;

166   if(years(tsim) gt firstYear,

168     $$iftheni.solve "%simType%" == "CompStat"

170     $$batinclude "solve.gms" gtap

172     $$else.solve

174     $$ifthen.calStatus %ifCal% == 1

176     $$batinclude "solve.gms" dynCal

178     $$else.calStatus
```

```
180         $$batinclude "solve.gms" dynGTAP
182         $$endif.calStatus
184         $$endif.solve
186     ) ;
188     display walras.l ;
189     put screen ;
190     put / ;
191     put "Walras: ", (walras.l/inScale) / ;
192     putclose screen ;
194     ts(tsim) = no ;
195 ) ;
197 $include "postsim.gms"
198 execute_unload "%outDir%/%simName%.gdx" ;
```

Appendix D. GTAP concordances

Table D.1. Regional concordance

<i>Name</i>	<i>GTAP Concordance</i>
1 Oceania	Australia (AUS), New Zealand (NZL), Rest of Oceania (XOC)
2 East Asia	China (CHN), Hong Kong (HKG), Japan (JPN), Korea (KOR), Mongolia (MNG), Taiwan (TWN), Rest of East Asia (XEA), Brunei Darussalam (BRN)
3 Southeast Asia	Cambodia (KHM), Indonesia (IDN), Laos (LAO), Malaysia (MYS), Philippines (PHL), Singapore (SGP), Thailand (THA), Viet Nam (VNM), Rest of Southeast Asia (XSE)
4 South Asia	Bangladesh (BGD), India (IND), Nepal (NPL), Pakistan (PAK), Sri Lanka (LKA), Rest of South Asia (XSA)
5 North America	Canada (CAN), United States of America (USA), Mexico (MEX), Rest of North America (XNA)
6 Latin America	Argentina (ARG), Bolivia (BOL), Brazil (BRA), Chile (CHL), Colombia (COL), Ecuador (ECU), Paraguay (PRY), Peru (PER), Uruguay (URY), Venezuela (VEN), Rest of South America (XSM), Costa Rica (CRI), Guatemala (GTM), Honduras (HND), Nicaragua (NIC), Panama (PAN), El Salvador (SLV), Rest of Central America (XCA), Dominican Republic (DOM), Jamaica (JAM), Puerto Rico (PRI), Trinidad and Tobago (TTO), Rest of Caribbean (XCB)
7 European Union (28)	Austria (AUT), Belgium (BEL), Cyprus (CYP), Czech Republic (CZE), Denmark (DNK), Estonia (EST), Finland (FIN), France (FRA), Germany (DEU), Greece (GRC), Hungary (HUN), Ireland (IRL), Italy (ITA), Latvia (LVA), Lithuania (LTU), Luxembourg (LUX), Malta (MLT), Netherlands (NLD), Poland (POL), Portugal (PRT), Slovakia (SVK), Slovenia (SVN), Spain (ESP), Sweden (SWE), United Kingdom (GBR), Bulgaria (BGR), Croatia (HRV), Romania (ROU)
8 Mid. East & N. Afr.	Bahrain (BHR), Iran (IRN), Israel (ISR), Jordan (JOR), Kuwait (KWT), Oman (OMN), Qatar (QAT), Saudi Arabia (SAU), Turkey (TUR), United Arab Emirates (ARE), Rest of Western Asia (XWS), Egypt (EGY), Morocco (MAR), Tunisia (TUN), Rest of North Africa (XNF)
9 Sub-Saharan Africa	Benin (BEN), Burkina Faso (BFA), Cameroon (CMR), Côte d'Ivoire (CIV), Ghana (GHA), Guinea (GIN), Nigeria (NGA), Senegal (SEN), Togo (TGO), Rest of Western Africa (XWF), Central Africa (XCF), South-Central Africa (XAC), Ethiopia (ETH), Kenya (KEN), Madagascar (MDG), Malawi (MWI), Mauritius (MUS), Mozambique (MOZ), Rwanda (RWA), Tanzania (TZA), Uganda (UGA), Zambia (ZMB), Zimbabwe (ZWE), Rest of Eastern Africa (XEC), Botswana (BWA), Namibia (NAM), South Africa (ZAF), Rest of South African Customs Union (XSC)
10 Rest of the world	Switzerland (CHE), Norway (NOR), Rest of EFTA (XEF), Albania (ALB), Belarus (BLR), Russian Federation (RUS), Ukraine (UKR), Rest of Eastern Europe (XEE), Rest of Europe (XER), Kazakhstan (KAZ), Kyrgyzstan (KGZ), Rest of Former Soviet Union (XSU), Armenia (ARM), Azerbaijan (AZE), Georgia (GEO), Rest of the World (XTW)

Table D.2. Concordance for sectors

<i>Name</i>	<i>GTAP Concordance</i>
1 Grains & crops (GrainsCrops)	Paddy rice (PDR), Wheat (WHT), Cereal grains, n.e.s. (GRO), Vegetables and fruits (V&F), Oil seeds (OSD), Sugar cane and sugar beet (C&B), Plant-based fibers (PFB), Crops, n.e.s. (OCR), Processed rice (PCR)
2 Livestock & meat products (MeatLstk)	Bovine cattle, sheep and goats, horses (CTL), Animal products n.e.s. (OAP), Raw milk (RMK), Wool, silk-worm cocoons (WOL), Bovine cattle, sheep and goat, horse meat products (CMT), Meat products n.e.s. (OMT)
3 Mining & extraction (Extraction)	Minerals n.e.s. (OMN), Forestry (FRS), Fishing (FSH), Coal (COA), Oil (OIL), Gas (GAS)
4 Processed food (ProcFood)	Vegetable oils and fats (VOL), Dairy products (MIL), Sugar (SGR), Food products n.e.s. (OFD), Beverages and tobacco products (B&T)
5 Textiles & clothing (TextWapp)	Textiles (TEX), Wearing apparel (WAP)
6 Light manufacturing (LightMnfc)	Leather products (LEA), Wood products (LUM), Paper products, publishing (PPP), Metal products (FMP), Motor vehicles and parts (MVH), Transport equipment n.e.s. (OTN), Manufactures n.e.s. (OMF)
7 Heavy manufacturing (HeavyMnfc)	Petroleum, coal products (P&C), Chemical, rubber, plastic products (CRP), Mineral products n.e.s. (NMM), Ferrous metals (I&S), Metals n.e.s. (NFM), Electronic equipment (ELE), Machinery and equipment n.e.s. (OME)
8 Utilities & construction (Util&Cons)	Electricity (ELY), Gas manufacture, distribution (GDT), Water (WTR), Construction (CNS)
9 Transport & communication (TransComm)	Trade (TRD), Transport n.e.s. (OTP), Sea transport (WTP), Air transport (ATP), Communication (CMN)
10 Other services (OthServices)	Insurance (ISR), Financial services n.e.s. (OFI), Business services n.e.s. (OBS), Recreation and other services (ROS), Public administration and defense, education, health services (OSG), Dwellings (DWE)